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Meson spectroscopy in AdS/CFT with flavour

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Abstract: We compute the meson spectrum of an $\mathcal{N}=2$ super Yang-Mills theory with fundamental matter from its dual string theory on $AdS_5 \times S^5$ with a D7-brane probe [1]. For scalar and vector mesons with arbitrary R-charge the spectrum is computed in closed form by solving the equations for D7-brane fluctuations; for matter with non-zero mass m_q it is discrete, exhibits a mass gap of order $m_q/\sqrt{g_s N}$ and furnishes representations of SO(5) even though the manifest global symmetry of the theory is only SO(4). The spectrum of mesons with large spin J is obtained from semiclassical, rotating open strings attached to the D7-brane. It displays Regge-like behaviour for $J \ll \sqrt{g_s N}$, whereas for $J \gg \sqrt{g_s N}$ it corresponds to that of two non-relativistic quarks bound by a Coulomb potential. Meson interactions, baryons and 'giant gauge bosons' are briefly discussed.

Keywords: D-branes, AdS-CFT and dS-CFT Correspondence.

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Contents

1. Introduction

The best-understood example of the AdS/CFT correspondence [2, 3, 4] (see [5] for a review) is the duality between IIB string theory on $AdS_5 \times S^5$ and $\mathcal{N} = \triangle$ SU(N) super Yang-Mills (SYM) theory, which can be motivated by analyzing the low-energy dynamics of a stack of N D3-branes [2] in Minkowski space. All matter fields in the gauge theory are in the adjoint representation. Of course, one can study the response of the theory to *external* sources in the fundamental representation by computing, for example, the Wilson loop. Using the AdS/CFT correspondence, this was done in [6] by studying a string with endpoints attached to the AdS boundary.

It was noted in [1] that by introducing k D7-branes into $AdS_5 \times S^5$, k flavours of *dynamical* quark (or, more precisely, k fundamental hypermultiplets) can be added to the gauge theory, breaking the supersymmetry to $\mathcal{N} = 2$. This can be understood by starting

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with a stack of N D3-branes and k parallel D7-branes in Minkowski space, which we represent by the array

$$\begin{array}{ll} \text{D3:} & 1\ 2\ 3\ \_\ \_\ \_\ \_\ \_\ \_ \\ \text{D7:} & 1\ 2\ 3\ 4\ 5\ 6\ 7\ \_\ \_\ . \end{array} \tag{1.1}$$

The hypermultiplets in the field theory description arise from the lightest modes of the 3-7 and 7-3 strings, hence their mass is given by $m_q = L/2\pi\alpha'$, where L is the distance between the D3- and the D7-branes in the 89-plane. If $g_s N \gg 1$ the D3-branes may be replaced (in the appropriate decoupling limit) by an $AdS_5 \times S^5$ geometry. If, in addition, $N \gg k$ then the back-reaction of the D7-branes on this geometry may be neglected and one is left, in the gravity description, with k D7-brane probes in $AdS_5 \times S^5$ that preserve eight supersymmetries [7].[1] In this paper we will often consider the case $k = 1$.

Although this system has similarities with those studied in [8, 9], it differs in some respects. In particular, the simplest case studied in these references involves a $\mathbb{Z}_2$-orientifold in flat space. As a consequence, the resulting gauge theory on the D3-branes has gauge group USp($2N$), and the corresponding dual gravitational description involves an orientifolded five-sphere. Note that the orientifold plane is necessary in flat space to cancel the D7-brane charge; the D7-branes in $AdS_5 \times S^5$, however, carry no net charge because they wrap a contractible cycle [1].

The $\mathcal{N} = 2$ theory with dynamical quarks possesses a rich spectrum of mesons, that is, quark-antiquark bound states. We perform a detailed computation of the mass spectrum of the different types of mesons using the dual gravity description. This was initiated in [10], where it was argued that the spectrum of scalar and vector mesons with arbitrary R-charge could be obtained from the fluctuations of the Born-Infeld (BI) fields on the D7-brane. Here, we solve the equations for the fluctuations in terms of hypergeometric functions, and then obtain the spectrum analytically. We obtain that for $m_q \neq 0$ it is discrete and exhibits a mass gap $m_{\text{gap}} \sim m_q/\sqrt{g_s N}$. Although the manifest global symmetry of the theory is SO(4), corresponding to rotations of the 4567-space, we find the unexpected result that the spectrum fills representations of a larger group, namely SO(5). This seems to be a large-N, large-$\sqrt{g_s N}$ effect that would be interesting to understand purely in field-theoretical terms. The effective low-energy action that describes the interactions between these mesons is given by the dimensional reduction of the D7-branes' action. We discuss some aspects of this effective action, in particular the scalar-scalar and vector-vector meson couplings, and find that their N-dependence agrees with the expectations from large-N gauge theory. Furthermore, the gauge invariance of the D7-branes' action is reflected in the fact that the coupling of the vector mesons to the other mesons is universal, being determined by the Yang-Mills coupling on the D7-branes. It is interesting to note that this is similar to QCD, where the observed universality of the ρ-meson couplings is attributed, in certain phenomenological models, to a hidden gauge symmetry of the low-energy lagrangian [22].

[1] These are Poincaré supersymmetries; if $L = 0$ then eight additional special conformal supersymmetries are preserved.

For large R-charge the BI modes resemble a classical, collapsed open string that follows a null geodesic along the compact directions of the D7-brane, thus providing an open string analogue of the closed strings recently considered in [15, 11]. For sufficiently large R-charge we expect the BI modes to expand into spherical D3-branes connected to the D7-brane by open strings; we will call these D3-branes 'giant gauge bosons'. We elaborate on all these aspects in the Discussion.

To obtain the spectrum of gauge theory mesons with four-dimensional spin $J > 1$, one has to compute the corresponding spectrum of open strings attached to the D7-brane in the $AdS_5 \times S^5$ background. Performing this exactly is difficult because of the non-trivial background. However, for $J \gg 1$ the spectrum can be obtained by treating the open strings semiclassically, in analogy to what was done recently for closed strings [11]. This involves solving the open string equations of motion with appropriate boundary conditions, which can be done numerically for arbitrary J. The result is perhaps surprising: for each fixed angular velocity of the string there is a series of solutions distinguished by the number of nodes $n = 0, 1, 2 \ldots$ of the string (see figure 1). The projection on the AdS boundary suggests a structure for the corresponding meson in which the two quarks are surrounded by concentric shells of gluons associated to the pieces of string between each two successive nodes.

Presumably the most stable solution, and the one that maximizes the spin for a fixed mass, corresponds to a string with no nodes. One possible way in which a self-intersecting string (i.e., one with $n > 0$ nodes) can decay is by breaking into an open string, with $n - 1$ nodes, and a closed string analogous to those considered in [11]; in the gauge theory this should correspond to the decay of an excited meson via the emission of a gluon shell. This process, however, is suppressed by a power of g_s. Self-intersecting strings could also be unstable under small fluctuations of the string taking one solution into another (with a different number of nodes). This happens in the case of an ordinary string being rotated under the effect of gravity, where, for a large number of nodes, the solution is such that the lowest point moves up and down rather than being static, as assumed here.[2]

For nodeless strings the spectrum can be obtained analytically in two particularly interesting regimes. For $1 \ll J \ll \sqrt{g_s N}$ the distance between the two quarks is small in comparison with the scale, $m_{\rm gap}^{-1}$, set by the mass of the lightest meson, and the string is much shorter than the AdS radius. It is therefore not surprising that in this regime we find Regge behaviour: $M \simeq \sqrt{J/\alpha'_{\rm eff.}}$, where the effective tension $(\alpha'_{\rm eff.})^{-1} \sim m_q^2/\sqrt{g_s N}$ can be understood as the string tension red-shifted to the point in AdS_5 where the string lies. Conversely, for $J \gg \sqrt{g_s N}$ the two quarks are far apart and the string is longer than the AdS radius. Here the result is $M \simeq m_q(2 - \kappa^4/4J^2)$ where κ is a constant. Both types of behaviour can be understood in terms of the quark-antiquark potential that we compute by studying a static open string attached to the D7-brane, in the spirit of [6]. In the first case the potential is linear in the separation between the quarks, whereas in the second case it is a Coulomb potential $V(\rho) = -\kappa^2/\rho$, and the quarks' motion is non-relativistic.

Since the $\mathcal{N} = 4$ theory does not have dynamical quarks, neither does it have dynamical baryons. It does have a baryon vertex, however, whose string dual was described in [12].

[2] We thank L. Freidel for this observation.

Once dynamical quarks are introduced in the theory there is a dynamical baryon. In the penultimate section we explain that its string dual is a D5-brane wrapping the five-sphere of $AdS_5 \times S^5$ and connected to the D7-brane by N fundamental strings, and discuss some of its properties.

2. The model

The massless modes of open strings with both ends on the N D3-branes give rise to an $\mathcal{N}=4$ vector multiplet consisting of the SU(N) vector bosons, four Weyl fermions and six scalars. $\mathcal{N}=4$ SYM is a conformal theory with R-symmetry group SO(6), under which the fermions and the scalars transform in the **4** and the **6** representations, respectively. All fields transform in the adjoint representation of SU(N) . This theory is dual to type-IIB string theory on $AdS_5 \times S^5$, whose metric may be written as

$$ds^2 = \frac{r^2}{R^2} ds^2 \left(\mathbb{E}^{(1,3)}\right) + \frac{R^2}{r^2} d\vec{Y} \cdot d\vec{Y} \,, \tag{2.1}$$

where $r = |\vec{Y}|$ and $R^2 = \sqrt{4\pi g_s N}\alpha'$. Note that, in the notation of the array (1.1), the Y^i, $i = 1, \ldots, 6$, parametrize the 456789-space. The four-dimensional conformal symmetry group SO(2, 4), and the R-symmetry group of the gauge theory correspond to the isometry groups of AdS_5 and S^5, respectively.

The addition of a D7-brane to the system, as in (1.1), breaks the supersymmetry to $\mathcal{N}=2$. The light modes coming from strings with one end on the D3-branes and the other one on the D7-brane give rise to an $\mathcal{N}=2$ hypermultiplet in the fundamental representation, whose field content is two complex scalar fields ϕ^m and two Weyl fermions $\psi_\pm$, of opposite chirality. If the D3-branes and the D7-brane overlap then the original SO(6) symmetry is broken to $\mathrm{SO}(4)\times\mathrm{SO}(2) \sim \mathrm{SU}(2)_R\times\mathrm{SU}(2)_L\times\mathrm{U}(1)_R$, where SO(4) and SO(2) rotate the 4567- and the 89-directions in (1.1), respectively. In this case the hypermultiplet is massless and the R-symmetry of the theory is $\mathrm{SU}(2)_R \times \mathrm{U}(1)_R$. The fields appear in representations $(j_1, j_2)_s$, where $j_{1,2}$ is the spin of $\mathrm{SU}(2)_{R,L}$ and s is the $\mathrm{U}(1)_R$ charge, in a normalization in which the supersymmetry generators transform as $(\frac{1}{2}, 0)_1$. With this convention, the two scalars transform in the $(\frac{1}{2}, 0)_0$ representation and the two fermions, $\psi_\pm$, are inert under $\mathrm{SU}(2)_R \times \mathrm{SU}(2)_L$ and transform with opposite, chirality-correlated $\mathrm{U}(1)_R$ charges ∓ 1.

If the D7-brane is separated from the D3-branes in the 89-plane then the hypermultiplet acquires a mass. It is known from field theory that the R-symmetry is then only $\mathrm{SU}(2)_R$, in agreement with the geometric interpretation: separating the D7-brane breaks the $\mathrm{U}(1)_R$ that acts on the 89-plane.

If we locate the D7-brane in the 89-plane at $|\vec{Y}| = L$, the induced metric is easily seen to be

$$ds^2 = \frac{\rho^2 + L^2}{R^2} ds^2 \left(\mathbb{E}^{(1,3)}\right) + \frac{R^2}{\rho^2 + L^2} d\rho^2 + \frac{R^2 \rho^2}{\rho^2 + L^2} d\Omega_3^2 \,, \tag{2.2}$$

where $\rho^2 = r^2 - L^2$ and Ω_3 are spherical coordinates in the 4567-space. We see that if $L = 0$ then this metric is exactly that of $AdS_5 \times S^3$. The AdS_5 factor suggests that the dual gauge theory should still be conformally invariant. This is indeed the case in the probe limit in which we are working: when $L = 0$ the quarks are massless and the theory is classically conformal. Quantum mechanically one finds that, in the large-N limit, the β-function for the 't Hooft coupling, $g_{YM}^2 N$, is proportional to the ratio k/N between the number of D7- and D3-branes, which goes to zero in the probe limit [1], as discussed in the Introduction. If $L \neq 0$ then the metric above becomes $AdS_5 \times S^3$ only asymptotically, i.e., for $\rho \gg L$, reflecting the fact that in the gauge theory conformal invariance is explicitly broken by the mass $m_q \propto L$ of the hypermultiplet, but is restored asymptotically at energies $E \gg m_q$. Note also that the radius of the three-sphere is not constant; in particular, it shrinks to zero at $\rho = 0$ (corresponding to $r = L$), at which point the D7-brane 'terminates' from the viewpoint of the projection on AdS_5 [1].

3. Meson spectrum (spin=$0, 1$, arbitrary R-charge)

In this section we compute the spectrum of scalar and vector mesons (and their superpartners) with arbitrary R-charge, and classify them in supersymmetric multiplets transforming in representations of the global symmetry $\mathrm{SU}(2)_R \times \mathrm{SU}(2)_L$. As we will also discuss, we actually find that the spectrum furnishes representations of a larger group, namely $\mathrm{SO}(5) \supset \mathrm{SU}(2)_R \times \mathrm{SU}(2)_L$. This involves an analysis of the quadratic part of the effective mesonic field theory. We then comment on the form of some of the couplings in the interacting sector. Finally, we describe the relation of the D7-brane modes to gauge theory operators in the conformal limit ($L = 0$).

The mesons in which we are interested in this section correspond to open string excitations of the D7-brane that are represented by scalar and gauge worldvolume fields carrying angular momentum on the three-sphere component of the D7-brane. Their dynamics is described by the action [13]

$$S_{D7} = -\mu_7 \int \mathrm{d}^8\xi \, \sqrt{-\det\left(P[G]_{ab} + 2\pi\alpha' F_{ab}\right)} + \frac{(2\pi\alpha')^2}{2}\mu_7 \int P[C^{(4)}] \wedge F \wedge F \,, \tag{3.1}$$

where the bulk metric G_{ab} was given in equation (2.1) and the relevant part of the Ramond-Ramond (RR) potential appearing in the Wess-Zumino term is given by

$$C^{(4)} = \frac{r^4}{R^4} \mathrm{d}x^0 \wedge \mathrm{d}x^1 \wedge \mathrm{d}x^2 \wedge \mathrm{d}x^3 \,. \tag{3.2}$$

Also, $\mu_7 = [(2\pi)^7 g_s \alpha'^4]^{-1}$ is the D7-brane tension and, as usual, P denotes the pullback of a bulk field to the brane's worldvolume.

As discussed before, this action has an $\mathrm{SU}(2)_R \times \mathrm{SU}(2)_L$ symmetry, corresponding to rotations of the S^3, and is the bosonic part of an action invariant under eight real supercharges ($\mathcal{N} = 2$ in four dimensions). The Wess-Zumino term breaks the symmetry that interchanges $\mathrm{SU}(2)_L$ and $\mathrm{SU}(2)_R$. In the field theory this is reflected in the fact that $\mathrm{SU}(2)_L$ commutes with the supercharges and $\mathrm{SU}(2)_R$ does not (as it is the R-symmetry group). The roles of $\mathrm{SU}(2)_{L,R}$ are reversed if we choose an anti-D7 brane, and this corresponds to a sign change in front of the Wess-Zumino term. We shall see how this is reflected in the masses of modes that transform differently under each SU(2).

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In the following subsections we explicitly compute the modes and their masses. However, it is useful to keep in mind how this works in the conformal case ($L = 0$), analyzed in [9]. When the theory is conformal we are interested in the conformal dimensions of operators dual to the D7-brane modes. These modes come in representations $(j_1, j_2)_s$ of $\mathrm{SU}(2)_R \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_R$, where $j_{1,2}$ denote the $\mathrm{SU}(2)_{R,L}$ spin and s the eigenvalue under the extra $\mathrm{U}(1)_R$ that appears in the $L = 0$ limit. The two scalar fields describing transverse oscillations of the D7-brane can be combined in a complex scalar Φ that, upon reduction on S^3, leads to a Kaluza-Klein tower of complex scalar fields Φ^ℓ transforming as $(\frac{\ell}{2}, \frac{\ell}{2})_2$, $\ell = 0, 1, 2, \ldots$. Similarly, the AdS components of the vector field on the D7-brane lead to a tower of AdS vectors A^ℓ that transform as $(\frac{\ell}{2}, \frac{\ell}{2})_0$, $\ell = 0, 1, 2, \ldots$. Finally, the gauge field components along S^3 lead to two different sets of real scalar fields A^ℓ_+ and A^ℓ_-, transforming as $(\frac{\ell-1}{2}, \frac{\ell+1}{2})_0$ and $(\frac{\ell+1}{2}, \frac{\ell-1}{2})_0$, $\ell = 1, 2, \ldots$, respectively.

All the above modes (plus the fermionic ones) organize themselves into short multiplets of the $\mathcal{N} = 2$ superconformal algebra,[3] and therefore their conformal dimensions are completely determined by their R-symmetry charges. The bosonic components of the supersymmetric multiplets are given by[4] $\{A^{\ell+1}_-, A^\ell, \Phi^\ell, A^{\ell-1}_+\}$. The dual multiplet of chiral operators in the gauge theory is generated by acting on a primary operator with the supersymmetry generators. This lowest dimension operator is a real scalar operator dual to $A^{\ell+1}_-$, hence transforming as $(\frac{\ell}{2} + 1, \frac{\ell}{2})_0$. Superconformal symmetry implies that the dimension of a scalar chiral primary operator of spin $\frac{\ell}{2}$ under $\mathrm{SU}(2)_R$ and R-charge q under $\mathrm{U}(1)_R$ is $\Delta = \ell + q/2$, so the operator dual to $A^{\ell+1}_-$ has $\Delta = \ell + 2$. Acting on it with QQ and $\bar{Q}\bar{Q}$ generates a complex scalar operator in the $(\frac{\ell}{2}, \frac{\ell}{2})_2$ representation, of dimension $\Delta = \ell + 3$; this is dual to Φ^ℓ. Similarly, acting with $Q\bar{Q}$ gives rise to a vector in the $(\frac{\ell}{2}, \frac{\ell}{2})_0$ representation, and with the same dimension, that is dual to A^ℓ. Finally, acting with $QQ\bar{Q}\bar{Q}$ leads to a real scalar operator of dimension $\Delta = \ell + 4$ that is dual to $A^{\ell-1}_+$ and hence transforms as $(\frac{\ell}{2} - 1, \frac{\ell}{2})_0$. Note that all operators in a given multiplet are in the same representation of $\mathrm{SU}(2)_L$ because the supersymmetry charges are invariant under $\mathrm{SU}(2)_L$.

Now we proceed to investigate the case $L \neq 0$.

3.1 Fluctuations of the scalar fields

As remarked above, the directions transverse to the D7-brane are chosen to be Y^5 and Y^6. The precise embedding is as follows:

$$Y^5 = 0 + 2\pi\alpha'\chi\,, \qquad Y^6 = L + 2\pi\alpha'\varphi\,, \tag{3.3}$$

with χ and φ the scalar fluctuations around the fiducial embedding.

To calculate the spectra of the worldvolume fields it suffices to work to quadratic order. For the scalars, we can write the relevant lagrangian density as

$$\mathcal{L} \simeq -\mu_7\sqrt{-\det g_{ab}}\left(1 + 2(R\pi\alpha')^2\frac{g^{cd}}{r^2}(\partial_c\chi\partial_d\chi + \partial_c\varphi\partial_d\varphi)\right). \tag{3.4}$$

[3] As representations of ordinary $\mathcal{N} = 2$ supersymmetry these are long multiplets, as corresponds to massive representations.

[4] This is the structure of the generic multiplet; for some low values of ℓ some of these components vanish.

All indices here denote worldvolume directions, and we have implicitly used static gauge. The induced metric, g_{ab}, factorises in such a way that the determinant appearing in equation (3.4) is independent of χ and φ. That is, there is no potential for these fields, as expected from BPS considerations. Therefore, to quadratic order, the lagrangian is completely independent of the fluctuations (as opposed to their derivatives), since we can drop them from the factor g^{cd}/r^2.

To more naturally incorporate the R-charge we change to spherical polar coordinates in the $(Y^1, \cdots, Y^4)$ directions,[5] with radius ρ. Then, given the previous discussion, we use $r^2 = \rho^2 + L^2$ and the induced metric (2.2) in the quadratic lagrangian. With this proviso, the two independent fluctuations are seen to appear identically and have the same equation of motion:

$$\partial_a \left(\frac{\rho^3 \sqrt{\det \tilde{g}}}{\rho^2 + L^2} g^{ab} \partial_b \Phi \right) = 0 \,. \tag{3.5}$$

Here, and in the following, Φ is used to denote either (real) fluctuation, and $\tilde{g}_{ij}$ is the metric on the round, unit three-sphere that, along with ρ, spans the $(Y^1, \cdots, Y^4)$ directions.

The equation of motion can be expanded as

$$\frac{R^4}{(\rho^2 + L^2)^2} \partial^\mu \partial_\mu \Phi + \frac{1}{\rho^3} \partial_\rho (\rho^3 \partial_\rho \Phi) + \frac{1}{\rho^2} \nabla^i \nabla_i \Phi = 0 \,, \tag{3.6}$$

where ∇_i is the covariant derivative on the three-sphere. We can use separation of variables to write the modes as

$$\Phi = \phi(\rho) e^{ik \cdot x} \mathcal{Y}^\ell (S^3) \,, \tag{3.7}$$

where $\mathcal{Y}^\ell(S^3)$ are the scalar spherical harmonics on S^3, which transform in the $(\frac{\ell}{2}, \frac{\ell}{2})$ representation of SO(4) and satisfy

$$\nabla^i \nabla_i \mathcal{Y}^\ell = -\ell(\ell+2) \mathcal{Y}^\ell \,. \tag{3.8}$$

Then equation (3.6) results in an equation for $\phi(\rho)$ that, after the redefinitions

$$\varrho = \frac{\rho}{L} \,, \qquad \bar{M}^2 = - \frac{k^2 R^4}{L^2} \,, \tag{3.9}$$

becomes

$$\partial_\varrho^2 \phi + \frac{3}{\varrho} \partial_\varrho \phi + \left(\frac{\bar{M}^2}{(1+\varrho^2)^2} - \frac{\ell(\ell+2)}{\varrho^2} \right) \phi = 0 \,. \tag{3.10}$$

One can show that this has solutions in terms of Legendre functions, a fact that will turn out be interesting later. Equivalently, it can be solved in terms of particular hypergeometric functions, which is the approach we will take here. If we first make the substitution

$$\phi(\varrho) = \varrho^\ell \, (1 + \varrho^2)^{-\alpha} \; P(\varrho) \,, \tag{3.11}$$

where

$$2\alpha = -1 + \sqrt{1 + \bar{M}^2} \geq 0 \,, \tag{3.12}$$

[5]The indices $a, b, c, \dots$ will still run over all the D7 coordinates. We shall use Latin indices $i, j, k, \dots$ to denote the coordinates on the S^3 (of unit radius) and Greek letters $\mu, \nu, \dots$ for directions parallel to the D3-brane.

and then make a further change of coordinates to $y = -\varrho^2$, equation (3.10) becomes the hypergeometric equation:

$$y(1-y)P''(y) + [c - (a+b+1)y]P'(y) - abP(y) = 0\,, \tag{3.13}$$

with $a = -\alpha$, $b = -\alpha + \ell + 1$ and $c = \ell + 2$. The form of the general solution then depends on the values taken by the parameters $\alpha \geq 0$ and $\ell \in \mathbb{N}$. With this in mind, and noting that the scalar fluctuations are real-valued and that $-\infty < y \leq 0$ (where the boundary lies at $y \to -\infty$ and $y = 0$ is a regular point), one finds that, up to a normalization constant, the only valid solution is [23]

$$P(y) = F(a\ ,\ b\ ;\ c\ ;\ y)\,, \tag{3.14}$$

where F is the standard hypergeometric function.

When rewritten in terms of ρ, the solution for ϕ is then

$$\phi(\rho) = \rho^\ell(\rho^2 + L^2)^{-\alpha}\ F\left(-\alpha\ ,\ -\alpha + \ell + 1\ ;\ \ell + 2\ ;\ -\frac{\rho^2}{L^2}\right). \tag{3.15}$$

Our criteria for validity of the solution are that it be real-valued, regular and small enough in amplitude to justify our use of the quadratic lagrangian. Furthermore, in order to be dual to a field theory state (a meson in this case), it must be normalizable [14]. Of these considerations, regularity at the origin and reality motivate the choice of solution made in equation (3.14). However, the hypergeometric function, in general an infinite series, may diverge as $\rho \to \infty$, making our use of the linearised equation of motion inconsistent and resulting in a non-normalizable mode. To find consistent normalizable solutions we must cause the series to terminate in such a way that its highest order term is suppressed by the prefactor $\rho^{\ell - 2\alpha}$. This can be ensured by setting

$$-\alpha + \ell + 1 = -n\,, \qquad n = 0, 1, 2, \dots\,, \tag{3.16}$$

in which case the hypergeometric function terminates at order $(-\rho^2/L^2)^n$, and $\phi \sim \rho^{-(\ell+2)}$ as $\rho \to \infty$. Our solution is then

$$\phi(\rho) = \frac{\rho^\ell}{(\rho^2 + L^2)^{n+\ell+1}}\ F\left(-(n+\ell+1)\ ,\ -n\ ;\ \ell+2\ ;\ -\rho^2/L^2\right). \tag{3.17}$$

The quantisation condition (3.16) gives

$$\bar{M}^2 = 4(n+\ell+1)(n+\ell+2)\,; \tag{3.18}$$

using this, and $M^2 = -k^2 = \bar{M}^2 L^2/R^4$, we derive the four-dimensional mass spectrum of scalar mesons to be

$$M_s(n,\ell) = \frac{2L}{R^2}\sqrt{(n+\ell+1)(n+\ell+2)}\,. \tag{3.19}$$

Normalizability of the modes results in a discrete spectrum with a mass scale set by L, the position of the D7-brane. For large R-charge, $\ell \gg n$, the scalar fluctuation is dominated by the prefactor $\rho^\ell/(\rho^2 + L^2)^\ell$, which peaks at $\rho_{\text{peak}} = L$ (i.e., $r = \sqrt{2}L$) and falls rapidly to zero on either side. The hypergeometric function modulates the fluctuation within this envelope. It is interesting to note that a massive particle orbiting the S^3 at a constant value of ρ approaches precisely $\rho_{\rm peak}$ as its angular momentum per unit mass increases; similarly, null geodesics lie exactly at $\rho_{\rm peak}$. In other words, in the large-ℓ limit, wave mechanics reduces to particle mechanics. Note that the relevant metric in these considerations is the *induced* metric (2.2) on the D7-brane, as opposed to the $AdS_5 \times S^5$ spacetime metric, because the modes in question are confined to propagate on the D7-brane worldvolume. For large ℓ they can be thought of as point-like, collapsed, massless open strings orbiting the S^3 along a null geodesic. These are potentially interesting because they are open string analogues of the closed strings that have recently led to the study of strings in pp-wave spacetimes [15]. We will come back to this in the Discussion.

Finally, the behaviour of the mode at infinity is related to the high-energy properties of the theory. At high energy we can ignore the effect of the mass of the quarks and the theory becomes conformal. This simply says that in the ultra-violet (UV) this theory flows to the one with $L = 0$ (in the large-N limit discussed above, where $1/N$ terms in the β-function are ignored). The $\rho \to \infty$ behaviour is then related to the UV operator of the lowest conformal dimension, Δ, that has the same quantum numbers as the meson [3, 4]. If the fields are canonically normalized then the behaviour at infinity is given by $\rho^{-\Delta}$ for the normalizable modes and $\rho^{\Delta-4}$ for the non-normalizable ones. In our case the kinetic terms in the lagrangian (3.4) are not canonically normalized, which means that the modes are multiplied by a common function of ρ and so behave as $\rho^{-\Delta+p}$ and $\rho^{\Delta-4+p}$, for some p. We can then obtain the conformal dimension from the difference between the exponents. For that purpose it is easier to consider an arbitrary real value of ℓ, for which the hypergeometric function appearing in equation (3.15) has the behaviour

$$F\left(\alpha_1\ ,\ \alpha_2\ ;\ \gamma\ ;\ -\frac{\rho^2}{L^2}\right) \simeq A\rho^{-2\alpha_1} + B\rho^{-2\alpha_2} \tag{3.20}$$

as $\rho \to \infty$. Assuming that $\alpha_2 > \alpha_1$, the first term corresponds to the non-normalizable part and the second one to the normalizable one. We then obtain a formula for the conformal dimension,

$$\Delta = 2 + \alpha_2 - \alpha_1\,, \tag{3.21}$$

that we will continue to use in the following subsection. In this particular case it gives

$$\Delta = \ell + 3\,. \tag{3.22}$$

3.2 Fluctuations of the gauge fields

The equations of motion for the gauge fields on the D7-brane, which follow from the action (3.1), are

$$\partial_a\left(\sqrt{-\det g_{cd}}\,F^{ab}\right) - \frac{4\rho(\rho^2+L^2)}{R^4}\varepsilon^{bjk}\partial_j A_k = 0\,, \tag{3.23}$$

where ε^{ijk} is a tensor density (i.e., it takes values ± 1) and we index the coordinates as before (see footnote 5). The second term in this equation is the contribution from the Wess-Zumino part of the action, proportional to the pullback of the RR five-form field strength, and is present only if b is one of the S^3 indices.

We can expand A_μ and A_ρ in (scalar) spherical harmonics on S^3, and the A_i in vector spherical harmonics. There are three classes of vector spherical harmonics. One is simply given by $\nabla_i \mathcal{Y}^\ell$. The other two, $\mathcal{Y}_i^{\ell,\pm}$, $\ell \geq 1$, transform in the $(\frac{\ell\mp1}{2}, \frac{\ell\pm1}{2})$ and satisfy

$$\nabla_i \nabla^i \mathcal{Y}_j^{\ell,\pm} - R_j^k \mathcal{Y}_k^{\ell,\pm} = -(\ell+1)^2 \mathcal{Y}_j^{\ell,\pm}\,, \tag{3.24}$$

$$\varepsilon_{ijk} \nabla_j \mathcal{Y}_k^{\ell,\pm} = \pm(\ell+1) \mathcal{Y}_i^{\ell,\pm}\,, \tag{3.25}$$

$$\nabla^i \mathcal{Y}_i^{\ell,\pm} = 0\,. \tag{3.26}$$

Here, $R_j^i = 2\delta_j^i$ is the Ricci tensor of an S^3 of unit radius. The square of the operator in the second equation equals minus the operator in the first one, which explains the relation between their eigenvalues. The modes containing $\mathcal{Y}_j^{\ell,\pm}$ do not mix with the others since they are in different representations of SO(4), so we have one type of mode given by

$$\text{type-I:} \qquad A_\mu = 0\,, \qquad A_\rho = 0\,, \qquad A_i = \phi_{\rm I}^{\pm}(\rho) e^{ik\cdot x} \mathcal{Y}_i^{\ell,\pm}(S^3)\,. \tag{3.27}$$

From the other modes we consider first those satisfying $\partial^\mu A_\mu = 0$. In that case one can see that the equations of motion for A_μ decouple from the others, so we have two further types of mode:

$$\text{type-II:} \qquad A_\mu = \zeta_\mu \phi_{\rm II}(\rho) e^{ik\cdot x} \mathcal{Y}^\ell(S^3)\,, \qquad k\cdot\zeta = 0\,, \qquad A_\rho = 0\,, \qquad A_i = 0\,; \tag{3.28}$$

and

$$\text{type-III:} \qquad A_\mu = 0\,, \qquad A_\rho = \phi_{\rm III}(\rho) e^{ik\cdot x} \mathcal{Y}^\ell(S^3)\,, \qquad A_i = \tilde{\phi}_{\rm III}(\rho) e^{ik\cdot x} \nabla_i \mathcal{Y}^\ell(S^3)\,. \tag{3.29}$$

Finally there are modes with $\partial^\mu A_\mu \neq 0$. For $k^2 = 0$ the equations do not lead to regular solutions whereas for $k^2 \neq 0$ the only independent such modes are those with polarizations $\zeta_\mu \sim k_\mu$. For these modes we can then always gauge away the A_μ components and we are left with modes of the type-III. This is equivalent to working in the gauge $\partial^\mu A_\mu = 0$ from the viewpoint of the gauge theory on the D7-brane. Now we should replace the modes in the equations of motion and obtain the spectrum.

For modes of type-I we only need the part with $b = j$ since the rest of the equations are satisfied identically. We obtain

$$\partial_\mu \partial^\mu A_i + \frac{1}{\rho}\partial_\rho \left(\frac{\rho(\rho^2+L^2)^2}{R^4}\partial_\rho A_i\right) + \frac{(\rho^2+L^2)^2}{R^4\rho^2}\left(\nabla_j\nabla^j A_i - R_i^j A_j\right) - \frac{4}{R^4}(\rho^2+L^2)\varepsilon_{ijk}\partial_j A_k = 0\,, \tag{3.30}$$

where we have used $\nabla^i A_i = 0$, as follows from equation (3.26). Using our ansatz and the properties of the vector spherical harmonics we obtain an equation for $\phi_{\rm I}(\rho)$ that, after the redefinition (3.9), reads

$$\frac{1}{\varrho}\partial_\varrho\left(\varrho(1+\varrho^2)^2\partial_\varrho \phi_{\rm I}^{\pm}(\varrho)\right) + \bar{M}^2 \phi_{\rm I}^{\pm}(\varrho) - (\ell+1)^2\frac{(1+\varrho^2)^2}{\varrho^2}\phi_{\rm I}^{\pm}(\varrho) \mp 4(\ell+1)(1+\varrho^2)\phi_{\rm I}^{\pm}(\varrho) = 0\,. \tag{3.31}$$

This equation is again equivalent to a Legendre equation. We can write the solutions regular at $\varrho = 0$ in terms of hypergeometric functions:

$$\phi_{\rm I}^{+}(\varrho) = \varrho^{\ell+1}(1+\varrho^2)^{-1-\alpha}\, F(\ell+2-\alpha\,,\ -1-\alpha\,;\ \ell+2\,;\ -\varrho^2)\,, \tag{3.32}$$

$$\phi_{\rm I}^{-}(\varrho) = \varrho^{\ell+1}(1+\varrho^2)^{-1-\alpha}\, F(\ell-\alpha\,,\ 1-\alpha\,;\ \ell+2\,;\ -\varrho^2)\,, \tag{3.33}$$

where we again define α as in (3.12). From normalizability considerations similar to those discussed for the scalars, we obtain the following spectra:

$$
\begin{aligned}
\bar{M}^2_{\mathrm{I},+} &= 4(n+\ell+2)(n+\ell+3)\,, \qquad && n\geq 0\,, \quad \ell \geq 1\,;\\
\bar{M}^2_{\mathrm{I},-} &= 4(n+\ell)(n+\ell+1)\,, \qquad && n\geq 0\,, \quad \ell \geq 1\,.
\end{aligned}
\tag{3.34}
$$

The behaviour of the modes at infinity is given by

$$
\phi^+_{\mathrm{I}}(\varrho) \sim \varrho^{-\ell-5}\,, \qquad \phi^-_{\mathrm{I}}(\varrho) \sim \varrho^{-\ell-1}\,. \tag{3.35}
$$

The conformal dimensions of the corresponding UV operators can be found using equation (3.21) and are $\Delta_+ = \ell{+}5$ for the mode that transforms in the $(\frac{\ell-1}{2}, \frac{\ell+1}{2})$, and $\Delta_- = \ell{+}1$ for the mode transforming in the $(\frac{\ell+1}{2}, \frac{\ell-1}{2})$.

For modes of type-II we only need the equation with $b = \mu$ since the rest are again identically satisfied. From the equation of motion we find

$$
\frac{R^4}{(\rho^2+L^2)^2}\partial^\nu\partial_\nu A_\mu + \frac{1}{\rho^3}\partial_\rho\left(\rho^3\partial_\rho A_\mu\right) + \frac{1}{\rho^2}\nabla^i\nabla_i A_\mu = 0\,, \tag{3.36}
$$

or

$$
\frac{\bar{M}^2}{(1+\varrho^2)^2}\phi_{\mathrm{II}}(\varrho) + \frac{1}{\varrho^3}\partial_\varrho\left(\varrho^3\partial_\varrho\phi_{\mathrm{II}}(\varrho)\right) - \ell(\ell+2)\frac{1}{\varrho^2}\phi_{\mathrm{II}}(\varrho) = 0\,. \tag{3.37}
$$

The solution is now

$$
\phi_{\mathrm{II}}(\varrho) = \varrho^{\ell}(1+\varrho^2)^{-\alpha}\; F(\ell+1-\alpha\;,\; -\alpha\;;\; \ell+2\;;\; -\varrho^2)\,, \tag{3.38}
$$

with spectrum

$$
\bar{M}^2_{\mathrm{II}} = 4(n+\ell+1)(n+\ell+2)\,, \qquad n\geq 0\,, \quad \ell\geq 0\,, \tag{3.39}
$$

and boundary behaviour

$$
\phi_{\mathrm{II}}(\varrho) \sim \varrho^{-\ell-2}\,. \tag{3.40}
$$

The associated UV conformal dimension is $\Delta = \ell + 3$.

Finally, for modes of type-III, the equation with $b = \mu$ gives a relation

$$
\ell(\ell+2)\tilde{\phi}_{\mathrm{III}}(\rho) = \frac{1}{\rho}\partial_\rho\left(\rho^3\phi_{\mathrm{III}}(\rho)\right)\,. \tag{3.41}
$$

For $\ell = 0$ this relation gives $\phi_{\mathrm{III}} \sim 1/\rho^2$, which is not regular at $\rho = 0$, so we must exclude this case. When $\ell \neq 0$, using this relation, the equations corresponding to $b = \rho$ and $b = j$ turn out to be equivalent, so we can write (after using (3.9))

$$
\partial_\varrho\left(\frac{1}{\varrho}\partial_\varrho\left(\varrho^3\phi_{\mathrm{III}}(\varrho)\right)\right) - \ell(\ell+2)\phi_{\mathrm{III}}(\varrho) - \frac{\bar{M}^2\varrho^2}{(1+\varrho^2)^2}\phi_{\mathrm{III}}(\varrho) = 0\,, \tag{3.42}
$$

with the solution

$$
\phi_{\mathrm{III}}(\varrho) = \varrho^{\ell-1}(1+\varrho^2)^{-\alpha}\, F(-\alpha+\ell+1\;,\; -\alpha\;;\; \ell+2\;;\; -\varrho^2)\,, \tag{3.43}
$$

spectrum

$$
\bar{M}^2_{\mathrm{III}} = 4(n+\ell+1)(n+\ell+2)\,, \qquad n\geq 0\,, \quad \ell\geq 1\,, \tag{3.44}
$$

and boundary behaviour

$$
\phi_{\mathrm{III}}(\varrho) \sim \varrho^{-\ell-3}\,. \tag{3.45}
$$

The conformal dimension of the associated UV operator is $\Delta = \ell + 3$.

3.3 Analysis of the spectrum

Summarising the previous results, the bosonic modes on the D7-brane give rise to the following mesonic spectrum:

$$\begin{array}{llll}
\text{2 scalars in the} & \left(\frac{\ell}{2},\frac{\ell}{2}\right) & \text{with } M_s^2(n,\ell) = \frac{4L^2}{R^4}(n+\ell+1)(n+\ell+2)\,, & n\geq 0\,, \quad \ell \geq 0,\ ; \\
\text{1 scalar in the} & \left(\frac{\ell}{2},\frac{\ell}{2}\right) & \text{with } M_{\mathrm{III}}^2(n,\ell) = \frac{4L^2}{R^4}(n+\ell+1)(n+\ell+2)\,, & n\geq 0\,, \quad \ell \geq 1\,; \\
\text{1 scalar in the} & \left(\frac{\ell-1}{2},\frac{\ell+1}{2}\right) & \text{with } M_{\mathrm{I},+}^2(n,\ell) = \frac{4L^2}{R^4}(n+\ell+2)(n+\ell+3)\,, & n\geq 0\,, \quad \ell \geq 1\,; \\
\text{1 scalar in the} & \left(\frac{\ell+1}{2},\frac{\ell-1}{2}\right) & \text{with } M_{\mathrm{I},-}^2(n,\ell) = \frac{4L^2}{R^4}(n+\ell)(n+\ell+1)\,, & n\geq 0\,, \quad \ell \geq 1\,; \\
\text{1 vector in the} & \left(\frac{\ell}{2},\frac{\ell}{2}\right) & \text{with } M_{\mathrm{II}}^2(n,\ell) = \frac{4L^2}{R^4}(n+\ell+1)(n+\ell+2)\,, & n\geq 0\,, \quad \ell \geq 0\,.
\end{array} \tag{3.46}$$

First note that there are no massless modes, i.e., there is a mass gap in the spectrum equal to the mass of the lightest meson, given by

$$m_{\mathrm{gap}} = 2\sqrt{2}\frac{L}{R^2} = 2m_q\sqrt{\frac{2\pi}{g_s N}}\,. \tag{3.47}$$

This means that in the regime $\sqrt{g_s N} \gg 1$, in which we are working, the meson mass is much smaller than the quark mass and so these mesons, together with the excitations of the $\mathcal{N}=4$ multiplet, dominate the physics at low energy. In perturbation theory one would find that $M \simeq 2m_q - E_b$, with a binding energy $E_b \sim (g_s N)^2$. At large 't Hooft coupling, however, we obtain that the theory is such that the binding energy almost cancels the rest energy of the quarks. This is clear from the bulk picture of meson 'formation', in which two strings of opposite orientation stretching from the D7-brane to the horizon (the quark-antiquark pair) join together to form an open string with both ends on the D7-brane (the meson); this resulting string is much shorter than the initial ones, and hence corresponds to a configuration with much lower energy.

Since the theory has $\mathcal{N}=2$ supersymmetry the mesons should fill (massive) supermultiplets. Since the supercharges commute with $\mathrm{SU}(2)_L$ all states in a given supermultiplet will be in the same representation of $\mathrm{SU}(2)_L$. The type of multiplet of interest here can be generated by applying the supercharges Q to a scalar state with spin $\frac{\ell}{2}$ under $\mathrm{SU}(2)_R$ that is annihilated by the $\bar{Q}$'s. Each of these multiplets contains an equal number, $8(\ell+1)$, of bosonic and fermionic states. Specifically, the generic ($\ell \geq 2$) multiplet consists of three real scalars and one vector in the $\frac{\ell}{2}$ of $\mathrm{SU}(2)_R$, two real scalars in the $\frac{\ell}{2} \pm 1$, and two Dirac fermions, one in the $\frac{\ell+1}{2}$ and one in the $\frac{\ell-1}{2}$. The bosonic content matches precisely the meson spectrum that we found, since

$$M_s(n,\ell) = M_{\mathrm{III}}(n,\ell) = M_{\mathrm{II}}(n,\ell) = M_{\mathrm{I},+}(n,\ell-1) = M_{\mathrm{I},-}(n,\ell+1)\,, \tag{3.48}$$

where, for the last two modes, we have shifted ℓ in such a way that all modes transform in the same $\frac{\ell}{2}$ representation of $\mathrm{SU}(2)_L$. If $\ell = 0$ the supermultiplet consists of two real scalars and one vector in the 0 of $\mathrm{SU}(2)_R$, one scalar in the 1, and one Dirac fermion in the

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1/2. If instead $\ell = 1$ then there are three real scalars and one vector in the 1/2, one scalar in the 3/2, one Dirac fermion in the 0 and another one in the 1. These two non-generic cases are also perfectly reproduced by the mode spectrum. For example, the fact that the type-III modes exist for all ℓ except for $\ell = 0$ agrees with the fact that the $\ell = 0$ multiplet contains one scalar field less than all other multiplets with $\ell > 0$.

Given the bosonic meson spectrum, supersymmetry allows us to obtain the fermionic spectrum as:

$$\begin{aligned} &1 \text{ Dirac fermion in the } \left(\frac{\ell+1}{2}, \frac{\ell}{2}\right) \text{ with } M_{F1}^2(n,\ell) = \frac{4L^2}{R^4}(n+\ell+1)(n+\ell+2)\,, \quad n,\ell \geq 0\,; \\ &1 \text{ Dirac fermion in the } \left(\frac{\ell}{2}, \frac{\ell-1}{2}\right) \text{ with } M_{F2}^2(n,\ell) = \frac{4L^2}{R^4}(n+\ell+2)(n+\ell+3)\,, \quad n,\ell \geq 0\,. \end{aligned} \tag{3.49}$$

Note that the spectrum exhibits a huge degeneracy, since all states with the same value of $\nu = n + \ell$ have the same mass. This suggests the existence of an extra, hidden symmetry that we now proceed to investigate.

We begin by noting that the first two scalar modes and the vector modes (see (3.46)) that have the same mass, $\bar{M}^2 = (\nu+1)(\nu+2)$, transform in the (reducible) representation

$$(0,0) \oplus \left(\frac{1}{2}, \frac{1}{2}\right) \oplus \cdots \oplus \left(\frac{\nu}{2}, \frac{\nu}{2}\right) \tag{3.50}$$

of $\mathrm{SU}(2)_R \times \mathrm{SU}(2)_L$. Perhaps surprisingly, this is precisely the decomposition in SO(4) representations of the representation of SO(5) of highest weight $[\nu, 0]$, which corresponds to scalar spherical harmonics on S^4. These are functions $\mathcal{Y}^\nu(S^4)$ that satisfy

$$\nabla^2_{S^4} \mathcal{Y}^\nu(S^4) = -\nu(\nu+3)\mathcal{Y}^\nu(S^4) = -[(\nu+1)(\nu+2) - 2]\mathcal{Y}^\nu(S^4)\,, \tag{3.51}$$

where $\nabla^2_{S^4}$ is the laplacian on S^4. We also see that the eigenvalue of the laplacian is, up to a constant, the meson mass. The rest of the bosonic modes, those that we called types III, (I,+) and (I,−), with a given value of ν, can also be assembled into a representation of SO(5) of highest weight[6] $[\nu, 1]$ (which corresponds to vector spherical harmonics on S^4), since this decomposes under $\mathrm{SU}(2)_R \times \mathrm{SU}(2)_L$ as

$$[\nu, 1] = \bigoplus_{\ell=1}^{\nu} \left[\left(\frac{\ell}{2}, \frac{\ell}{2}\right) \oplus \left(\frac{\ell+1}{2}, \frac{\ell-1}{2}\right) \oplus \left(\frac{\ell-1}{2}, \frac{\ell+1}{2}\right) \right]. \tag{3.52}$$

Note, however, that the modes in question do not have the same mass, since $M_{\mathrm{III}}(n,\ell)$, $M_{\mathrm{I},+}(n,\ell)$ and $M_{\mathrm{I},-}(n,\ell)$ only coincide if we shift the ℓ's as in (3.48). We therefore conclude that the spectrum furnishes representations of SO(5), but that not all states in the same irreducible representation have the same mass. This means that SO(5) is not an exact symmetry (i.e., it does not commute with the hamiltonian operator). This was to be expected, since an SO(5) symmetry would imply a symmetry under the interchange of $\mathrm{SU}(2)_L$ with $\mathrm{SU}(2)_R$, but this is not present here, as is manifest from the different masses of the modes (I,+) and (I,−).

[6]Note that for type-III modes we cannot use a $[\nu, 0]$ representation, since these modes only exist for $\ell \geq 1$.

The appearance of SO(5) and the fact that it only implies a mass degeneracy for certain modes can be understood more geometrically as follows. The induced metric (2.2) on the D7-brane is conformally equivalent to that of $\mathbb{E}^{(1,3)} \times S^4$, since it can be written as

$$ds^2 = \frac{L^2}{R^2}(1+\varrho^2)\, ds^2(\hat{g})\,, \tag{3.53}$$

where

$$ds^2(\hat{g}) = ds^2(\mathbb{E}^{(1,3)}) + \frac{R^4}{4L^2}\left[\frac{4}{(1+\varrho^2)^2}(d\varrho^2 + \varrho^2 d\Omega_3^2)\right]. \tag{3.54}$$

This is the metric on $\mathbb{E}^{(1,3)} \times S^4$, written using the coordinate ϱ defined in (3.9). We have factored it in such a way that the expression in square brackets is the metric of a unit four-sphere in stereographic coordinates.[7] The induced worldvolume metric has this property because the $AdS_5 \times S^5$ metric (2.1) itself is conformally flat: in terms of new coordinates $\vec{Z} = R^2\vec{Y}/r^2$, it takes the form

$$ds^2 = \frac{R^2}{|\vec{Z}|^2}\left[ds^2(\mathbb{E}^{(1,3)}) + d\vec{Z}\cdot d\vec{Z}\right], \tag{3.55}$$

and the D7-brane embedding equation, $r = L$, becomes the equation of a four-sphere. Put yet another way, the inversion that takes $\vec{Y}$ into $\vec{Z}$ transforms a four-plane into a four-sphere.

The conformal factor in (3.53) depends on one of the coordinates of the four-sphere, and therefore it explicitly breaks the SO(5) symmetry of (3.54) down to SO(4). If the theory on the D7-brane were conformally invariant then we could ignore the conformal factor and conclude that the theory possesses an exact SO(5) symmetry. Of course, the D7-brane theory is not conformally invariant, but for some modes the effect of the conformal factor on the *quadratic* part of the action can be compensated by a field redefinition, which explains the SO(5) mass degeneracy of the corresponding sector of the *free* spectrum. Let us illustrate this for the first two scalar modes in (3.46).

Starting again from the lagrangian (3.4), but now using the induced metric in the form (3.53), we obtain

$$\mathcal{L} \simeq -\frac{\mu_7 L^4 (2\pi\alpha')^2}{2R^4}(1+\varrho^2)^2 \sqrt{\hat{g}}\, \hat{g}^{ab}\, \partial_a\varphi\partial_b\varphi\,, \tag{3.56}$$

where $\hat{g}$ denotes the determinant of the $\mathbb{E}^{(1,3)} \times S^4$ metric $\hat{g}_{ab}$ (3.54). We only write one of the scalars explicitly, since the (quadratic) lagrangian for the other, χ, is exactly the same. As expected, the factor $(1+\varrho^2)^2$ seems to break the SO(5) symmetry but, if we define a new field as

$$\tilde{\varphi} = (1+\varrho^2)\varphi\,, \tag{3.57}$$

we can rewrite the lagrangian as

$$\mathcal{L} \simeq -\frac{\mu_7 L^4 (2\pi\alpha')^2}{2R^4}\sqrt{\hat{g}}\left[\hat{g}^{ab}\,\partial_a\tilde{\varphi}\partial_b\tilde{\varphi} + \frac{2L^2}{R^4}\left(2\varrho^2\tilde{\varphi}^2 - \varrho(1+\varrho^2)\partial_\varrho(\tilde{\varphi}^2)\right)\right]. \tag{3.58}$$

[7] The relation between ϱ and the familiar azimuthal angle θ is $\varrho = \tan(\theta/2)$.

The first term is manifestly invariant under SO(5) but the others are not. Surprisingly, however, if we integrate the final term by parts (within the action), we find that all of the ϱ-dependence of the square bracket can be extracted into the prefactor $\sqrt{\hat{g}}$, and the lagrangian simplifies to

$$\mathcal{L} \simeq -\frac{\mu_7 L^4 (2\pi\alpha')^2}{R^4} \sqrt{\hat{g}} \left(\frac{1}{2} \hat{g}^{ab}\, \partial_a \tilde{\varphi} \partial_b \tilde{\varphi} + \frac{4L^2}{R^4} \tilde{\varphi}^2 \right), \tag{3.59}$$

which is manifestly SO(5)-invariant, as claimed. Furthermore, by expanding $\tilde{\varphi}$ in scalar spherical harmonics $\mathcal{Y}^\nu(S^4)$ and integrating over the S^4, we obtain a non-interacting, (3+1)-dimensional effective lagrangian that consists of a sum of individual quadratic lagrangians, one for each mode. Up to numerical pre-factors, the lagrangian for the ν-th mode takes the form

$$\mathcal{L} \simeq -\mu_7 R^4 (2\pi\alpha')^2 \left(\partial_\mu \tilde{\varphi} \partial^\mu \tilde{\varphi} + \frac{4L^2}{R^4} [\nu(\nu+3)+2] \tilde{\varphi}^2 \right). \tag{3.60}$$

We thus see that the mass of the ν-th scalar is $M^2 = (4L^2/R^4)(\nu+1)(\nu+2)$, as before. Furthermore, as suggested by the fact that equation (3.10) has Legendre function solutions, the wavefunctions agree, after a change of variables, with those found earlier in (3.7) and (3.17). In particular, the hypergeometric functions of equation (3.17) combine with the $\mathcal{Y}(S^3)$'s of equation (3.7) to produce the $\mathcal{Y}(S^4)$'s we expand in here.

An analogous calculation can be done for the vector mesons with the same result, namely that the quadratic lagrangian can be rewritten in a manifestly SO(5)-invariant way, but this is not possible for the rest of the modes. This explains the mass degeneracy of the scalar and vector modes noted above, and the absence of such a degeneracy in the rest of the spectrum. Let us again stress that this partial degeneracy is a feature of the free spectrum, since it follows from certain properties of the quadratic D7-brane action.

Although SO(5) is not a true symmetry, the fact that the spectrum furnishes representations of SO(5) does have a predictive power, since it implies that, once a certain meson is present in the spectrum, all other mesons necessary to build the corresponding SO(5) representation must also be present (albeit not necessarily with the same mass). The regularity of the spectrum suggests that one should be able to take this issue further and uncover other 'symmetries' of the spectrum, including perhaps a spectrum-generating algebra. We leave further investigation of this interesting subject for the future.

We wish to close this section by noting that the unexpected appearance of SO(5) discussed here will presumably have an analogue for other Dp-branes in AdS: an inversion will map the directions parallel to the Dp-brane and orthogonal to the D3-branes into a sphere. This could have interesting applications in the context of the AdS/dCFT correspondence [16, 17, 7].

3.4 Meson interactions

The discussions in the previous subsections amount to an analysis of the non-interacting sector of the effective meson field theory. Here we wish to comment briefly on their interactions, gained by expanding the D7-brane action (3.1) to higher order in the scalar and vector fields. This (3+1)-dimensional theory is simply the KK reduction of the worldvolume theory over the S^3 and along the radial direction.

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The simplest scalar interaction to consider is cubic in the fields and is contained in the lagrangian already written in equation (3.4) (using the metric (2.2)):

$$\mathcal{L} \simeq -\mu_7 \sqrt{-\det g_{ab}} \left(1 + 2(R\pi\alpha')^2 \frac{g^{cd}}{r^2} (\partial_c \chi \partial_d \chi + \partial_c \varphi \partial_d \varphi) \right) . \tag{3.61}$$

In the above, we truncated this to quadratic order by neglecting the field-dependence of the factor g^{cd}/r^2. If, instead, we expand g^{cd}/r^2 to first order in the fields, the terms of interest are

$$\begin{aligned} \mathcal{L} \simeq -\mu_7 (2\pi\alpha')^2 \rho^3 \sqrt{\tilde{g}} \Bigg(& \frac{R^4}{2(\rho^2+L^2)^2} (\partial_\mu \chi \partial^\mu \chi + \partial_\mu \varphi \partial^\mu \varphi) + \frac{1}{2} (\partial_\rho \chi)^2 + \frac{1}{2} (\partial_\rho \varphi)^2 + \\ & + \frac{1}{2\rho^2} \tilde{g}^{ij} (\partial_i \chi \partial_j \chi + \partial_i \varphi \partial_j \varphi) - \frac{4R^4 L\pi\alpha'}{(\rho^2+L^2)^3} \varphi (\partial_\mu \chi \partial^\mu \chi + \partial_\mu \varphi \partial^\mu \varphi) \Bigg) , \end{aligned} \tag{3.62}$$

recalling that $\tilde{g}_{ij}$ is the metric on the unit three-sphere. The classical equations of motion can be applied to remove the radial and angular derivatives, by integrating by parts in the action. To do so, we first decompose each field in terms of its four-dimensional spectrum, as follows:

$$\chi = \sum_a \frac{\tilde{\phi}_{n_a \ell_a}(\varrho)}{L^{\gamma_a}} \mathcal{Y}^{\ell_a}(S^3)\, h_a(x^\mu) , \qquad \varphi = \sum_b \frac{\tilde{\phi}_{n_b \ell_b}(\varrho)}{L^{\gamma_a}} \mathcal{Y}^{\ell_b}(S^3)\, f_a(x^\mu) , \tag{3.63}$$

where the sum over a, b runs over all values of n, ℓ and the other quantum numbers implicit on $\mathcal{Y}^\ell$. The radial and angular factors each satisfy their respective equations of motion, (3.10) and (3.8). Since the radial parts, given in (3.17), have been written in terms of the dimensionless coordinate $\varrho = \rho/L$, we have extracted their dimensionful content in the factors $L^{-\gamma_a}$, where $\gamma_a = 2(n_a+1)+\ell_a$. The h's and f's are then the KK towers of mesonic fields in whose interactions we are interested.

In performing the KK reduction, the integrals over ϱ and S^3 will produce dimensionless functions of the four quantum numbers that specify each KK mode. After substituting the field decompositions (3.63) into the lagrangian, each quadratic term will contain a product of two radial and two angular factors. Their orthogonality properties result in the corresponding integrals being proportional to $\delta_{n_a n_b}$ and $\delta_{\ell_a \ell_b}$, respectively. Their product therefore provides a factor proportional to δ_{ab} that we shall denote by $\mathcal{K}_a \delta_{ab}$ (no sum). Conversely, each term in the cubic lagrangian will contain a product of three radial and three angular factors and, in general, no such simplication of the corresponding integrals is possible. Hence, we will denote them by $\mathcal{R}_{abc}$.

With these considerations, after applying the equations of motion, the lagrangian is written

$$\begin{aligned} \mathcal{L} = {} & \mu_7 R^4 (2\pi\alpha')^2 \sum_a \frac{1}{L^{2\gamma_a}} \mathcal{K}_a \left(-\frac{1}{2} (\partial_\mu h_a \partial^\mu h_a + \partial_\mu f_a \partial^\mu f_a) - \frac{1}{2} M_a^2 (h_a^2 + f_a^2) \right) + \\ & + \mu_7 R^4 (2\pi\alpha')^3 \frac{2}{L} \sum_{a,b,c} \frac{1}{L^{\gamma_a+\gamma_b+\gamma_c}} \mathcal{R}_{abc}\, f_a (\partial_\mu h_b \partial^\mu h_c + \partial_\mu f_b \partial^\mu f_c) . \end{aligned} \tag{3.64}$$

The mass parameters appearing in this expression depend on the n's and ℓ's through equation (3.19). We are now in a position to normalize the fields so as to leave the quadratic lagrangian in canonical form, in order to arrive at an expression that we can interpret as a (3+1)-dimensional effective theory of the mesons. Noting that $\mu_7 R^4 (2\pi\alpha')^2 = N/(8\pi^4)$, the required field redefinitions are clearly

$$\bar{h}_a = \frac{1}{2\pi^2 L^{\gamma_a}} \sqrt{\frac{N\mathcal{K}_a}{2}}\, h_a \,, \tag{3.65}$$

and a similar expression for $\bar{f}_a$. Comparison with (3.63) shows that the $\bar{h}$'s and $\bar{f}$'s have the same dimensions as χ and φ, namely (length)$^{-1}$, appropriate to scalar fields in four dimensions. If we then substitute these new fields into the cubic part of the lagrangian (3.64):

$$\mathcal{L}^{(3)} = (2\pi)^3 \sqrt{\frac{2}{N}} \frac{\alpha'}{L} \sum_{a,b,c} \frac{\mathcal{R}_{abc}}{\sqrt{\mathcal{K}_a \mathcal{K}_b \mathcal{K}_c}}\, \bar{f}_a (\partial_\mu \bar{h}_b \partial^\mu \bar{h}_c + \partial_\mu \bar{f}_b \partial^\mu \bar{f}_c) \,, \tag{3.66}$$

we can read off the dimensionful cubic coupling strength as

$$g_{\Phi(\partial\Phi)^2} \sim \frac{1}{\sqrt{N}} \frac{\alpha'}{L} \sim \frac{1}{\sqrt{N}} \frac{1}{m_q} \,, \tag{3.67}$$

using Φ to denote a generic scalar. A similar calculation using the expansion of the lagrangian to fourth order reveals

$$g_{\Phi^4} \sim \frac{1}{\lambda} \frac{1}{N} \qquad g_{\Phi^2(\partial\Phi)^2} \sim \frac{1}{N} \frac{1}{m_q^2} \,, \qquad \text{and} \qquad g_{(\partial\Phi)^4} \sim \frac{\lambda}{N} \frac{1}{m_q^4} \tag{3.68}$$

for the quartic couplings, where $\lambda = g_s N$ is the 't Hooft coupling. Note that, from the field theory viewpoint, the dependence on the quark mass m_q follows from dimensional analysis and the dependence on N agrees with a large-N argument [18]. The dependence on the 't Hooft coupling is a prediction of our calculations in AdS.

Now we turn to the vector mesons. They correspond to the type-II A_μ components of the eight-component worldvolume vector. To find the couplings of their self-interactions one may expand the action in equation (3.1) to higher order to produce terms of the form F^4. Alternatively, as we will do, one may consider the case of a non-abelian worldvolume theory, where standard gauge interactions would appear at leading order. The leading gauge term of the Born-Infeld lagrangian is proportional to Tr F^2, and contains the quadratic and leading (in an α' expansion) cubic and quartic interactions, since the non-abelian field strength contains commutators of the gauge fields: $F_{ab} = \partial_a A_b - \partial_b A_a + i[A_a, A_b]$. Considering terms involving the vector mesons only, the relevant lagrangian is

$$\begin{aligned}\mathcal{L} = -\frac{\mu_7 (2\pi\alpha')^2}{4} \rho^3 \sqrt{\tilde{g}} \Bigg(& \frac{R^4}{(\rho^2+L^2)^2} \operatorname{Tr} \tilde{F}_{\alpha\beta} \tilde{F}^{\alpha\beta} + 2 \operatorname{Tr} \partial_\rho A_\alpha \partial_\rho A^\alpha + \frac{2}{\rho^2} \tilde{g}^{ij} \operatorname{Tr} \partial_i A_\alpha \partial_j A^\alpha + \\ & + \frac{R^4}{(\rho^2+L^2)^2} \Big(2i \operatorname{Tr} \tilde{F}_{\alpha\beta} [A^\alpha, A^\beta] - \operatorname{Tr} [A_\alpha, A_\beta][A_\gamma, A_\delta] \Big) \Bigg) \,, \end{aligned} \tag{3.69}$$

where $\tilde{F}_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha$.

The gauge fields take values in the adjoint representation of the gauge group, U(k), and can be decomposed, in terms of its k^2 generators, as $A_\alpha = A_\alpha^{(s)}\,\tau_s$. The functional coefficients $A_\alpha^{(s)}$ each take the same form (3.28) as the abelian type-II fields discussed earlier and can therefore be decomposed, in turn, as was done for the scalar fields in (3.63). In what follows it will not be necessary to write the U(k) fields in terms of their generators, so we use the decomposition

$$A_\alpha = \sum_a \frac{\tilde{\phi}_{n_a \ell_a}(\varrho)}{L^{\gamma_a}}\, \mathcal{Y}^{\ell_a}(S^3)\, W_{a\,\alpha}\,, \tag{3.70}$$

where $\tilde{\phi}_{n_a\ell_a}(\varrho)$ satisfies the equation of motion (3.37), and we have introduced an effective four-dimensional vector field $W_{a\,\alpha}$ for each mode of A_α.

We can now substitute (3.70) into (3.69) and perform the KK reduction. The resulting integrals in the quadratic part of (3.69) will be identical to those in (3.64), but those in the cubic and quartic parts will not. In a similar notation to that used above, we denote them by $\tilde{\mathcal{R}}_{abc}$ and $\tilde{\mathcal{R}}_{abcd}$. Then, after applying the classical equations of motion, (3.69) becomes

$$\begin{aligned}\mathcal{L} = \mu_7 R^4 (2\pi\alpha')^2 \Bigg(&\sum_a \frac{1}{L^{2\gamma_a}} \mathcal{K}_a \left(-\frac{1}{4}\,\mathrm{Tr}\; G_{a\,\alpha\beta} G_a^{\alpha\beta} - \frac{1}{2} M_a^2\; \mathrm{Tr}\; W_{a\,\alpha} W_a^\alpha \right) + \\ &+ \frac{i}{2} \sum_{abc} \frac{1}{L^{\gamma_a+\gamma_b+\gamma_c}} \tilde{\mathcal{R}}_{abc}\; \mathrm{Tr}\; G_{a\,\alpha\beta} [W_b^\alpha, W_c^\beta] - \\ &- \frac{1}{4} \sum_{abcd} \frac{1}{L^{\gamma_a+\gamma_b+\gamma_c+\gamma_d}} \tilde{\mathcal{R}}_{abcd}\; \mathrm{Tr}\, [W_{a\,\alpha}, W_{b\,\beta}] \left[W_c^\alpha, W_d^\beta \right] \Bigg)\,,\end{aligned} \tag{3.71}$$

using $G_{a\,\alpha\beta} = \partial_\alpha W_{a\,\beta} - \partial_\beta W_{a\,\alpha}$. By comparison with (3.64) one can see that the field redefinition required to put (3.71) in standard form is exactly the same as that required for the scalars, (3.65). By counting fields, it is then clear that after making the redefinitions the couplings of the cubic and quartic interactions are

$$g_{W^2(\partial W)} \sim \frac{1}{\sqrt{N}}\,, \qquad \text{and} \qquad g_{W^4} \sim \frac{1}{N}\,, \tag{3.72}$$

respectively, with no dependence on the quark mass or the 't Hooft coupling. This again matches the large-N expectations [18].

3.5 Field/operator correspondence

As discussed before, close to the boundary the D7-brane is embedded as an $AdS_5 \times S^3$ subspace and the theory is conformal in the large-N limit. The asymptotic behaviour of the modes determines the conformal dimension of the lowest-dimension operator that, in the conformal theory, has the same quantum numbers as the meson. From the previous results we have seen that the conformal dimensions are: $\Delta_{s,\mathrm{III},\mathrm{II}} = \ell + 3$ for the scalar and vector modes transforming in the $(\frac{\ell}{2}, \frac{\ell}{2})$ and coming from Φ, A_ρ and A_μ; $\Delta_+ = \ell + 5$ for the scalar mode transforming in the $(\frac{\ell-1}{2}, \frac{\ell+1}{2})$; and $\Delta_- = \ell + 1$ for the scalar mode transforming in the $(\frac{\ell+1}{2}, \frac{\ell-1}{2})$. This is in precise agreement with what was described at the beginning of this section. In particular, notice that what we call here A_μ and A_ρ comprise the AdS_5 vector that we previously called A. This gives a check on the large-ρ behaviour of the modes we have found.

Furthermore, we can find the operators in the conformal field theory that are dual to these modes. First, all operators dual to D7-brane modes must contain at least two hypermultiplet fields. Moreover, it is easy to see that they must contain *exactly* two of these fields if they correspond to single-particle states. With this in mind, the chiral primary operators dual to the modes A^1_- are uniquely determined by symmetry. Indeed, these modes transform in the $(1,0)_0$ representation. The dual operators must have the same quantum numbers, i.e., they must be invariant under $\mathrm{SU}(2)_L \times \mathrm{U}(1)_R$ and transform as a triplet under $\mathrm{SU}(2)_R$; in addition they must have conformal dimension $\Delta = \ell + 1 = 2$. It is easy to see that the unique possibility for the dual of A^1_- is the $\mathrm{SU}(2)_R$-triplet of operators

$$\mathcal{O}^I = \bar{\phi}^m \sigma^I_{mn} \phi^n \,, \tag{3.73}$$

where $I = 1, 2, 3$ and σ^I are the Pauli matrices. A set of analogous operators was also found in [17], in the context of a D5-brane in $AdS_5 \times S^5$, to be the chiral primaries dual to the lowest-angular momentum modes on the brane.

As usual in AdS/CFT, we expect the higher-ℓ chiral primaries, $\mathcal{O}_\ell$, dual to A^ℓ_- to be constructed as follows. Consider the multiplet of operators

$$\mathcal{X}_\ell = Y^{(i_1} \cdots Y^{i_{\ell-1})} \tag{3.74}$$

where Y^i $(i = 1, \ldots, 4)$ are a subset of the six adjoint scalars of the $\mathcal{N} = 4$ multiplet, and the parentheses stand for traceless symmetrization of the indices. This set of operators transforms under $\mathrm{SU}(2)_R \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_R$ in the $(\frac{\ell-1}{2}, \frac{\ell-1}{2})_0$ representation, which can be composed with the $(1,0)_0$ to obtain the $(\frac{\ell+1}{2}, \frac{\ell-1}{2})_0$ representation. We therefore expect that the operator of dimension $\Delta = \ell + 3$, dual to the A^ℓ_- fields, takes the form

$$\mathcal{O}_\ell = \bar{\phi}^m \sigma_{mn} \, \mathcal{X}_\ell \, \phi^n \,, \tag{3.75}$$

with the SO(4) indices implicit in σ_{mn} and $\mathcal{X}$ appropriately composed to obtain the $(\frac{\ell+1}{2}, \frac{\ell-1}{2})_0$ representation.

The operators in each chiral multiplet dual to the rest of the D7-brane modes can be obtained from the chiral primaries described above by acting on the latter with the appropriate combinations of supercharges.

For large but finite N, the operators $\mathcal{O}_\ell$ are only independent for $\ell = 0, \ldots, N-1$. In the last section we discuss how this truncation may be reproduced on the AdS side.

4. Meson spectrum (large spin, no R-charge)

We now turn to the spectrum of mesons with four-dimensional spin J. As mentioned in the Introduction, an exact calculation would require quantizing open strings attached to the D7-brane, which is not feasible because of the non-trivial background. However, for large J the spectrum can be obtained from classical, rotating open strings. In order to minimize the energy, the string end points will be attached to the D7-brane at the minimum possible value of r (from the AdS_5 viewpoint), that is, at $r = L$.

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We expect two different limiting regimes to appear depending on how the proper size δ of the string compares to the AdS radius $R \sim (g_s N \alpha'^2)^{1/4}$. If $\delta \ll R$ then the curvature of AdS is irrelevant and hence the spectrum should be that found in flat space, i.e., we should find $E \sim \sqrt{J}$. Since in flat space $\delta \sim \sqrt{J\alpha'}$, this regime should occur when $J \ll \sqrt{g_s N}$. If instead $J \gg \sqrt{g_s N}$, then the AdS curvature becomes important and the spectrum is drastically modified; we will see that it agrees with that of two non-relativistic quarks weakly bound by a Coulomb potential.

To study the rotating string we will work with the Nambu-Goto form of the action:

$$S = \int \mathrm{d}\tau \, L(X, X', \dot{X}) = -\frac{1}{2\pi\alpha'} \int \mathrm{d}\tau \, \mathrm{d}\sigma \sqrt{(\dot{X} \cdot X')^2 - \dot{X}^2 X'^2} \, . \tag{4.1}$$

Here τ and σ parametrize the string worldsheet Σ, $X^M(\tau, \sigma)$ specify its embedding in spacetime, the dots and the primes denote differentiation with respect to τ and σ, respectively, and the scalar products are taken with respect to the $AdS_5 \times S^5$ metric. The string configurations we are seeking lie at a point on S^5 and within a two-plane in $\mathbb{E}^3$, so the relevant part of the spacetime metric is

$$ds^2 = \frac{R^2}{z^2} \left(-dt^2 + d\rho^2 + \rho^2 d\theta^2 + dz^2 \right) . \tag{4.2}$$

The coordinate z is related to the coordinate r in equation (2.1) by $z = R^2/r$, so the AdS boundary is at $z = 0$. The D7-brane extends from the boundary $z = 0$ to a maximum value $z = z_{\rm D7} = R^2/L$. As mentioned before, fixing $z_{\rm D7}$ amounts to fixing the unique scale m_q in the gauge theory. Recall that the mass of the dynamical quarks is $m_q = L/2\pi\alpha' = R^2/2\pi\alpha' z_{\rm D7}$.

We fix the time reparametrization invariance of the string worldsheet by identifying $t \equiv \tau$. In addition we set $\theta = \omega t$ for constant ω, which means that the string rotates in the $\rho\theta$-plane and hence carries a non-zero spin. Under these circumstances the string action becomes

$$S = -\frac{R^2}{2\pi\alpha'} \int \mathrm{d}\tau \mathrm{d}\sigma \, \frac{1}{z^2} \sqrt{(1 - \omega^2 \rho^2) \left(\rho'^{\,2} + z'^{\,2} \right)} \, , \tag{4.3}$$

where $\rho(\sigma)$ and $z(\sigma)$ specify the time-independent string profile.[8] In order to eliminate the dependence of the equations of motion on ω, it is convenient to work with the rescaled, dimensionless coordinates

$$\tilde{\rho} = \omega \rho \, , \qquad \tilde{z} = \omega z \, , \tag{4.4}$$

in terms of which the action becomes

$$S = -\frac{R^2 \omega}{2\pi\alpha'} \int \mathrm{d}\tau \mathrm{d}\sigma \, \frac{1}{\tilde{z}^2} \sqrt{(1 - \tilde{\rho}^2) \left(\tilde{\rho}'^{\,2} + \tilde{z}'^{\,2} \right)} \, . \tag{4.5}$$

Similarly, the energy and the spin are

$$E = \omega \frac{\partial L}{\partial \omega} - L = \frac{R^2 \omega}{2\pi\alpha'} \int \mathrm{d}\sigma \, \frac{1}{\tilde{z}^2} \sqrt{\frac{\tilde{\rho}'^{\,2} + \tilde{z}'^{\,2}}{1 - \tilde{\rho}^2}} \, , \tag{4.6}$$

[8] The action (4.3) is consistent in the sense that any solution of its equations of motion automatically provides a solution of those of (4.1).

$$J = \frac{\partial L}{\partial \omega} = \frac{R^2}{2\pi\alpha'} \int \mathrm{d}\sigma \, \frac{\tilde{\rho}^2}{\tilde{z}^2} \sqrt{\frac{\tilde{\rho}'^{\,2} + \tilde{z}'^{\,2}}{1-\tilde{\rho}^2}} \,. \tag{4.7}$$

Note that the condition that the string endpoints are attached to the D7-brane at z_{D7} translates into the Dirichlet boundary condition $\tilde{z}|_{\partial\Sigma} = \omega z_{\mathrm{D7}}$.

Two convenient choices to fix the worldspace reparametrization invariance of the string are $\tilde{\rho} \equiv \sigma$ and $\tilde{z} \equiv \sigma$. In the first case we are left with an equation of motion for $\tilde{z}(\tilde{\rho})$:

$$\frac{\tilde{z}''}{1+\tilde{z}'^2} + \frac{2}{\tilde{z}} - \frac{\tilde{\rho}\tilde{z}'}{1-\tilde{\rho}^2} = 0 \,, \tag{4.8}$$

whereas in the second case we have an equation for $\tilde{\rho}(\tilde{z})$:

$$\frac{\tilde{\rho}''}{1+\tilde{\rho}'^2} - \frac{2}{\tilde{z}}\tilde{\rho}' + \frac{\tilde{\rho}}{1-\tilde{\rho}^2} = 0 \,. \tag{4.9}$$

As we will see, each choice breaks down at a discrete number of points along the string.

As is well-known, the equations of motion for an open string must be supplemented with the boundary conditions

$$\left. \frac{\partial L}{\partial (X')^M} \delta X^M \right|_{\partial\Sigma} = 0 \tag{4.10}$$

that ensure that the action is stationary. Since $\delta\tilde{z}|_{\partial\Sigma} = 0$ and $\delta\tilde{\rho}|_{\partial\Sigma}$ is arbitrary (due to the Neumann boundary condition), we must impose $(\partial L/\partial \tilde{\rho}')_{\partial\Sigma} = 0$, that is,

$$\left. \frac{\tilde{\rho}'}{\tilde{z}^2} \sqrt{\frac{1-\tilde{\rho}^2}{\tilde{\rho}'^2 + \tilde{z}'^2}} \right|_{\partial\Sigma} = 0 \,. \tag{4.11}$$

It follows that either $\tilde{\rho}'|_{\partial\Sigma} = 0$, which means that the string ends orthogonally on the D7-brane, or $\tilde{\rho}|_{\partial\Sigma} = 1$, which means that the endpoints of the string move at the speed of light.[9] The second condition cannot correspond to a bound state of two hypermultiplet quarks because it cannot describe a string with both endpoints on the D7-brane. Indeed, by expanding $\tilde{z}(\tilde{\rho})$ for $\tilde{\rho} \lesssim 1$, substituting it into (4.8) and applying this boundary condition, we find

$$\tilde{z} \simeq \omega z_{\mathrm{D7}} - \frac{2}{3\omega z_{\mathrm{D7}}}(1-\tilde{\rho})^2 + \cdots \,. \tag{4.12}$$

This means that $\tilde{z}(\tilde{\rho})$ has a maximum at $\tilde{\rho} = 1$. From equation (4.8) we see that if $\tilde{z}' = 0$ then $\tilde{z}'' < 0$, that is, $\tilde{z}(\tilde{\rho})$ can have no minima but only maxima, and therefore it can have only one maximum. It follows that if we start at one endpoint of the string with the boundary conditions $\tilde{z} = \omega z_{\mathrm{D7}}$ and $\tilde{\rho} = 1$ then the string profile $\tilde{z}(\tilde{\rho})$ decreases monotonically as $\tilde{\rho}$ decreases away from $\tilde{\rho} = 1$, and therefore the other string endpoint cannot be attached to the D7-brane at the same value $\tilde{z} = \omega z_{\mathrm{D7}}$. It is possible, however, that a string with these boundary conditions can extend from the D7-brane to the AdS boundary at $\tilde{z} = 0$.

We conclude that the appropriate boundary condition for our purposes is that the string ends orthogonally on the D7-brane, i.e., that $\tilde{\rho}'|_{\partial\Sigma} = 0$; note for later use that in the

[9] The same result is obtained by using the Polyakov action and imposing the constraints.

gauge $\tilde{\rho} = \sigma$ this corresponds to $\tilde{z}'|_{\partial\Sigma} \to \infty$. The speed of the endpoints of the string is determined by the actual solution and in general is subluminal. This is easily understood by approximating a small region around either endpoint by flat space. In this case it is clearly possible to get a solution with an arbitrary velocity, $v < 1$, by boosting the solution in which a static string ends on a D-brane.

To obtain the spectrum $E(J)$ we must solve for the string profile in the $\tilde{z}\tilde{\rho}$-plane and substitute it into equations (4.6) and (4.7); this determines $E(\omega)$ and $J(\omega)$, and hence the spectrum in parametric form. Equation (4.8) can be integrated numerically for arbitrary ω by starting at a point $\tilde{\rho} = 0$, $\tilde{z} = \tilde{z}_0$ with the boundary condition $\tilde{z}'(0) = 0$; this is motivated by the expectation that the solution must be symmetric around $\tilde{\rho} = 0$ and hence that the only maximum of $\tilde{z}(\tilde{\rho})$ must be at $\tilde{\rho} = 0$. To allow for the fact that the string may double-back across $\tilde{\rho} = 0$ we will allow $\tilde{\rho}$ to take positive and negative values. The numerical integration then results in a profile for half of the string, the other half being its mirror image. The motion of the string is due to the revolution of the full profile around the point $\tilde{\rho} = 0$, with angular velocity ω. The results are perhaps surprising: for each value of ω there is a series of solutions distinguished by the number of nodes $n = 0, 1, 2, \ldots$, or points at which the string intersects itself (see figure 1). Remarkably, all of these solutions can be continued past the position of the D7-brane (at which $\tilde{z} = \omega z_{\rm D7}$ and $\tilde{\rho}' = 0$) to the AdS boundary at $\tilde{z} = 0$. The extended solutions obtained in this way compute Wilson loops corresponding to two quarks of infinite mass moving around each other with angular velocity ω. A brief check of the numerical results is that they exhibit the following behaviour, in agreement with the equations of motion (4.8) and (4.9). First, $\tilde{z}(\tilde{\rho})$ has a unique maximum at $\tilde{\rho} = 0$. Second, at all inflexion points, at which $\tilde{z}'' = 0$, we have that $\mathrm{sgn}(\tilde{z}') = \mathrm{sgn}(\tilde{\rho})$. Third, if $\tilde{\rho}' = 0$ then $\mathrm{sgn}(\tilde{\rho}'') = -\,\mathrm{sgn}(\tilde{\rho})$, that is, $\tilde{\rho}(\tilde{z})$ has only maxima for $\tilde{\rho} > 0$ and only minima for $\tilde{\rho} < 0$. Finally, there is one exception to this last behaviour, occuring at $\tilde{z} = 0$. There one finds that $\tilde{\rho}' \to 0$ but that $\mathrm{sgn}(\tilde{\rho}'') = \mathrm{sgn}(\tilde{\rho})$, meaning that $\tilde{\rho}(0) > 0$ is a local minimum and $\tilde{\rho}(0) < 0$ is a local maximum. Each of these observations can be verified from figure 1.

As explained in the Introduction, the projection on the AdS boundary of these solutions suggests a structure for the corresponding dual mesons in which the two quarks are surrounded by concentric shells of gluons associated to the pieces of string between each two successive nodes. The deeper the nodes are in AdS space the larger the radius of the shell should be.

The most stable solutions are presumably those without nodes, since those with nodes can break at the self-intersection points, corresponding to the decay of an excited meson via the emission of a gluon shell. We will concentrate on the $n = 0$ solutions in what follows. The numerical results for the spectrum are displayed in figure 3. Analytical results can be obtained in the two limiting regimes $J \ll \sqrt{g_s N}$ and $J \gg \sqrt{g_s N}$, which, as we shall see, correspond to $\omega \to \infty$ and $\omega \to 0$, respectively.

4.1 Large angular velocity, or $J \ll \sqrt{g_s N}$

Let us start with the case $\omega \to \infty$. One can see from the nodeless solution in figure 1 that $\tilde{z} > \omega z_{\rm D7}$, in which case the second term in (4.8) may be negligible. If we strictly

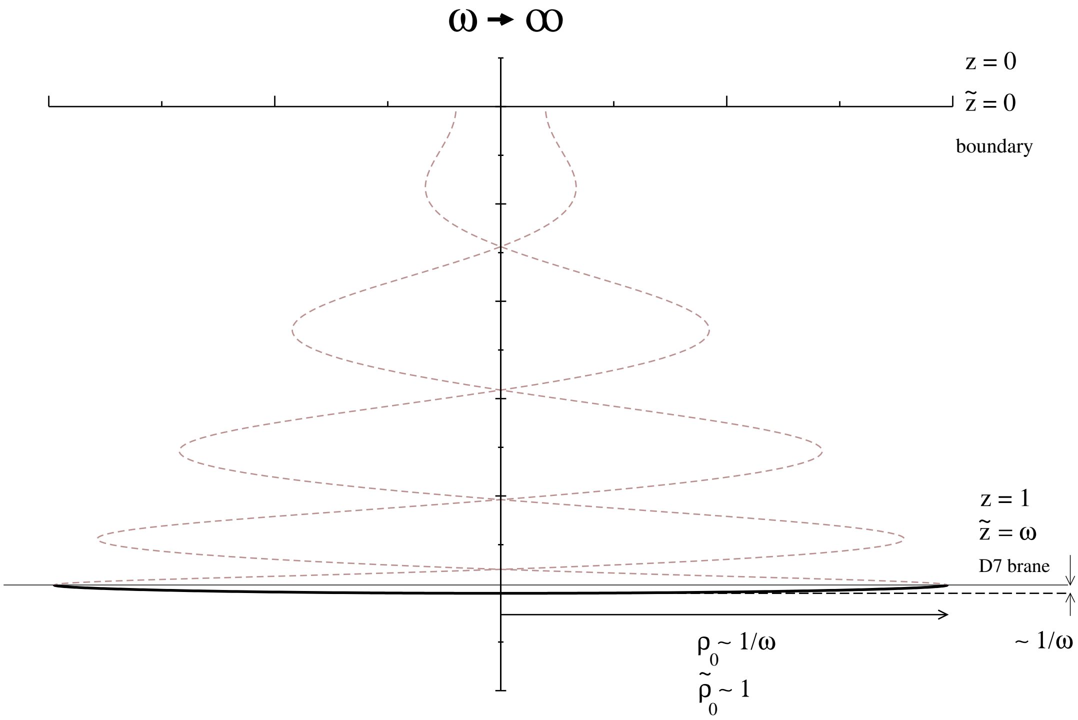

Figure 1: Solution for large angular velocity, or equivalently for $J \ll \sqrt{g_s N}$. For illustrative purposes we have set $z_{\mathrm{D7}} = 1$. The thick solid line corresponds to a solution with no nodes, attached to a D7-brane represented by the thin solid line. The physical size of the string is very small compared to the AdS radius (as is apparent from its coordinate size in the 'physical coordinates' ρ and z), and the solution is well approximated by a straight string at $z = z_{\mathrm{D7}} = 1$, as would be the case in flat space. The continuation of the nodeless solution along the dashed line provides solutions with $n = 1, 2, 3$ or 4 nodes, depending on which maximum of $\tilde{z}(\tilde{\rho})$ the corresponding D7-brane (not shown in the figure) is placed at. The continuation beyond the last maximum that terminates at the AdS boundary is dual to a Wilson loop corresponding to two infinitely heavy external quarks orbiting around each other with angular velocity ω.

ignore it then the only solution satisfying the appropriate boundary conditions is a constant solution $\tilde{z}(\tilde{\rho}) = \omega z_{\mathrm{D7}}$, which is precisely what one would expect in the flat space limit, and is a suggestion that $\omega \to \infty$ corresponds to the $J \ll \sqrt{g_s N}$ case. Note that since $0 < |\tilde{\rho}| < 1$, after rescaling back we have $0 < |\rho| < 1/\omega \to 0$, namely a very short string insensitive to the AdS curvature. Corrections to this solution come from considering the $2/\tilde{z}$ term in (4.8) and so are of order $1/\omega z_{\mathrm{D7}}$. Substituting the ansatz

$$\tilde{z} = \omega z_{\mathrm{D7}} + \frac{1}{\omega z_{\mathrm{D7}}} f(\tilde{\rho}) \tag{4.13}$$

into (4.8) and keeping terms of order $1/\omega z_{\mathrm{D7}}$ we get an equation for f,

$$f'' + 2 - \frac{\tilde{\rho} f'}{1 - \tilde{\rho}^2} = 0\,, \tag{4.14}$$

that must be supplemented with the boundary conditions $f(0) \simeq 0, f'(0) = 0$. This is solved by elementary methods, with the result

$$f(\tilde{\rho}) = \frac{1}{2}\left(\tilde{\rho}^2 + \arcsin^2(\tilde{\rho})\right). \tag{4.15}$$

The coordinate $\tilde{\rho}$ has a maximum value $\tilde{\rho}_0$, where the string connects to the D7-brane. This point is determined by the condition $\tilde{z}'(\tilde{\rho}_0) \to \infty$. Since

$$\tilde{z}' = \frac{1}{\omega z_{\text{D7}}}\left(\tilde{\rho} + \frac{\arcsin(\tilde{\rho})}{\sqrt{1-\tilde{\rho}^2}}\right), \tag{4.16}$$

we then have $\tilde{\rho}_0 = 1$, which means that the endpoints of the string move at the speed of light. This is a result of the approximation used here; recall, however, that they must move at subluminal speed in the exact solution, a fact confirmed by the numerical analysis. By substituting the solution (4.13), (4.15) into equations (4.6) and (4.7) we obtain the energy and the spin to leading order in $1/\omega z_{\text{D7}}$:

$$E \simeq \frac{R^2}{2\alpha'\omega z_{\text{D7}}^2}, \qquad J \simeq \frac{R^2}{4\alpha'\omega^2 z_{\text{D7}}^2}. \tag{4.17}$$

Note that since $\omega \gg 1$ we have $J \ll \sqrt{g_s N}$, as anticipated. Eliminating ω and writing z_{D7} in terms of L or, equivalently, m_q, we find

$$E \simeq \frac{R}{z_{\text{D7}}}\sqrt{\frac{J}{\alpha'}} = \frac{L}{R}\sqrt{\frac{J}{\alpha'}} = \frac{\sqrt{2}\pi^{3/4} m_q}{(g_s N)^{1/4}}\sqrt{J}. \tag{4.18}$$

We conclude that for $J \ll \sqrt{g_s N}$ the meson masses follow a Regge trajectory with an effective tension

$$\tau_{\text{eff.}} = \frac{1}{2\pi\alpha'_{\text{eff.}}} = \frac{E^2}{2\pi J} = m_q^2\sqrt{\frac{\pi}{g_s N}} = \frac{1}{2\pi\alpha'}\frac{R^2}{z_{\text{D7}}^2}. \tag{4.19}$$

As the last expression shows, this tension can be simply understood as a proper tension $1/2\pi\alpha'$ at $z = z_{\text{D7}}$, which is then red-shifted as seen by a boundary observer.

4.2 Small angular velocity, or $J \gg \sqrt{g_s N}$

We now turn to the case $\omega \to 0$. Since the string is now attached to the D7-brane very close to the AdS boundary (in the $\tilde{z}$ coordinate) and rotates very slowly, we expect it to be well approximated by the solution $\tilde{\rho}(\tilde{z})$ for $\omega = 0$ that determines the static potential between two quarks. This static solution is well-known [6], and in our coordinates takes the form

$$\tilde{\rho}_{\text{st.}}(\tilde{z}) = \int_{\tilde{z}}^{\tilde{z}_0} \mathrm{d}x\, \frac{x^2}{\sqrt{\tilde{z}_0^4 - x^4}}. \tag{4.20}$$

Note that the factors of ω implicit in the tilded coordinates cancel from this expression, so it is valid in the $\omega \to 0$ limit. To compute the first correction we set $\tilde{\rho} = \tilde{\rho}_{\text{st.}} + \delta\tilde{\rho}$ and substitute this into (4.9) to obtain

$$\delta\tilde{\rho}'' - \frac{2}{\tilde{z}}\left(1 + 3\tilde{\rho}'^2_{\text{st.}}\right)\delta\tilde{\rho}' = -\tilde{\rho}_{\text{st.}}\frac{1 + \tilde{\rho}'^2_{\text{st.}}}{1 - \tilde{\rho}^2_{\text{st.}}}, \tag{4.21}$$

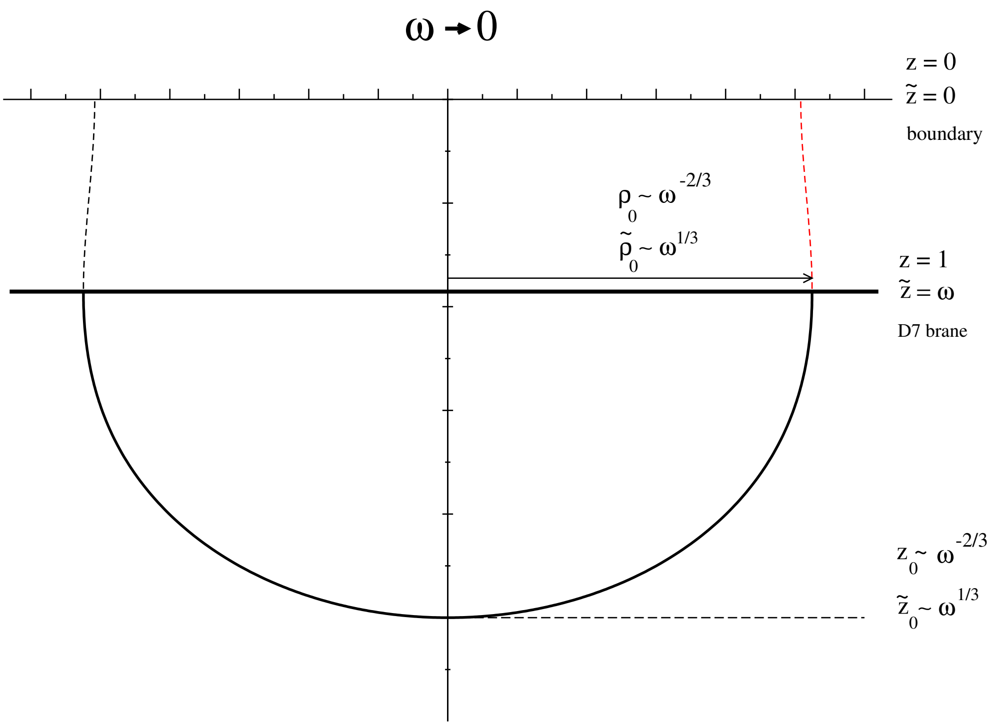

Figure 2: Solution for small angular velocity ($J \gg \sqrt{g_s N}$). Again, the dashed line represents the continuation of the solution beyond the D7-brane. Note that although in coordinates $(\tilde{\rho}, \tilde{z})$ the string is small it actually is very large in the physical coordinates (ρ, z).

where we have made use of the fact that $\tilde{\rho}_{\rm st.}$ satisfies equation (4.9) without the third term. The boundary conditions for $\delta\tilde{\rho}$ are $\delta\tilde{\rho}(\tilde{z}_0) = \delta\tilde{\rho}'(\tilde{z}_0) = 0$.

The solution is easily found to be

$$\delta\tilde{\rho}(\tilde{z}) = -\int_{\tilde{z}_0}^{\tilde{z}} \mathrm{d}x\, \frac{Q(\tilde{z}) - Q(x)}{Q'(x)}\, \tilde{\rho}_{\rm st.}(x)\, \frac{1 + \tilde{\rho}'^2_{\rm st.}(x)}{1 - \tilde{\rho}^2_{\rm st.}(x)}\,, \tag{4.22}$$

where the function

$$Q(\tilde{z}) \equiv \int^{\tilde{z}} \mathrm{d}x\, \frac{x^2}{(\tilde{z}_0^4 - x^4)^{3/2}} \tag{4.23}$$

has an expression (that we will not need) in terms of elliptic functions.

The relationship between $\tilde{z}_0$, ω and $z_{\rm D7}$ is determined by the condition that the string ends orthogonally on the D7-brane, that is,

$$\tilde{\rho}'(\omega z_{\rm D7}) = \tilde{\rho}'_{\rm st.}(\omega z_{\rm D7}) + \delta\tilde{\rho}'(\omega z_{\rm D7}) = 0\,. \tag{4.24}$$

This implies that

$$\int_{\omega z_{\rm D7}}^{\tilde{z}_0} \mathrm{d}x\, \frac{\sqrt{\tilde{z}_0^4 - x^4}}{x^2}\, \frac{\tilde{\rho}_{\rm st.}(x)}{1 - \tilde{\rho}^2_{\rm st.}(x)} = 1 - \frac{\omega^4 z_{\rm D7}^4}{\tilde{z}_0^4}\,. \tag{4.25}$$

We know from [6] that $\tilde{\rho}_{\rm st.}(0) = \mathcal{C}\tilde{z}_0$, where

$$\mathcal{C} \equiv \int_1^\infty \frac{\mathrm{d}y}{y^2\sqrt{y^4 - 1}} = \frac{\sqrt{2}\pi^{3/2}}{\Gamma(1/4)^2} \simeq 0.599 \tag{4.26}$$

and we have argued that in the $\omega \to 0$ limit this should receive a small correction, which is such that $\tilde{\rho}(\omega z_{\rm D7}) \simeq \tilde{\rho}_{\rm st.}(0)$ (see, e.g., figure 2). To leading order in $\tilde{z}_0$, as $\tilde{z}_0 \to 0$, we can then use $\tilde{\rho}_{\rm st.}(x) \simeq \tilde{\rho}_{\rm st.}(0)$ and neglect $\tilde{\rho}^2_{\rm st.}(x)$ in equation (4.25). Then, further assuming that $\omega z_{\rm D7} \ll \tilde{z}_0$, we obtain

$$1 \simeq \tilde{z}_0^2 \tilde{\rho}_{\rm st.}(0) \int_{\omega z_{\rm D7}}^{\tilde{z}_0} \frac{{\rm d}x}{x^2} \simeq \frac{\mathcal{C}\tilde{z}_0^3}{\omega z_{\rm D7}} \quad \Rightarrow \quad \omega \simeq \frac{\mathcal{C}\tilde{z}_0^3}{z_{\rm D7}} \,. \tag{4.27}$$

Note from this that $\omega z_{\rm D7} \ll \tilde{z}_0$ as $\tilde{z} \to 0$, which makes the approximation self-consistent. The energy and angular momentum can be computed within the same approximation, i.e., to leading order in $\tilde{z}_0$, as follows. Using the parametrization $\tilde{z} = \sigma$, the string's energy is written as, c.f. equation (4.6),

$$E = \frac{R^2\omega}{\pi\alpha'} \int_{\omega z_{\rm D7}}^{\tilde{z}_0} \frac{{\rm d}\tilde{z}}{\tilde{z}^2} \sqrt{\frac{1+\tilde{\rho}'^2}{1-\tilde{\rho}^2}} \simeq \frac{R^2\omega}{\pi\alpha'} \left(1 + \frac{\mathcal{C}^2\tilde{z}_0^2}{2}\right) \int_{\omega z_{\rm D7}}^{\tilde{z}_0} \frac{{\rm d}\tilde{z}}{\tilde{z}^2} \sqrt{1+\tilde{\rho}'^2_{\rm st.}} \left(1 + \frac{\tilde{\rho}'_{\rm st.}\delta\tilde{\rho}'}{1+\tilde{\rho}'^2_{\rm st.}}\right) \tag{4.28}$$

using $\tilde{\rho}(\tilde{z}) \simeq \tilde{\rho}_{\rm st.}(0) = \mathcal{C}\tilde{z}_0$. The first term in the integral can be split up as

$$\begin{aligned} \left(\frac{1}{\omega z_{\rm D7}} - \frac{1}{\tilde{z}_0}\right) + \int_{\omega z_{\rm D7}}^{\tilde{z}_0} {\rm d}\tilde{z} \left(\frac{\sqrt{1+\tilde{\rho}'^2_{\rm st.}}}{\tilde{z}^2} - \frac{1}{\tilde{z}^2}\right) &\simeq \frac{1}{\omega z_{\rm D7}} + \frac{1}{\tilde{z}_0} \left(\int_0^1 {\rm d}y \left[\frac{1}{y^2\sqrt{1-y^4}} - \frac{1}{y^2}\right] -1\right) \\ &= \frac{1}{\omega z_{\rm D7}} - \frac{\mathcal{C}}{\tilde{z}_0} \,, \end{aligned} \tag{4.29}$$

meaning that

$$\begin{aligned} E &\simeq \frac{R^2\omega}{\pi\alpha'} \left(1 + \frac{\mathcal{C}^2\tilde{z}_0^2}{2}\right) \left[\left(\frac{1}{\omega z_{\rm D7}} - \frac{\mathcal{C}}{\tilde{z}_0}\right) + \int_{\omega z_{\rm D7}}^{\tilde{z}_0} \frac{{\rm d}\tilde{z}}{\tilde{z}^2} \frac{\tilde{\rho}'_{\rm st.}\delta\tilde{\rho}'}{\sqrt{1+\tilde{\rho}'^2_{\rm st.}}}\right] \\ &= \frac{R^2}{\pi\alpha' z_{\rm D7}} \left[1 - \frac{\mathcal{C}^2\tilde{z}_0^2}{2} + \mathcal{O}(\tilde{z}_0^4)\right], \end{aligned} \tag{4.30}$$

where the integral we have not evaluated explicitly contributes a subdominant $\mathcal{O}(\tilde{z}_0^4)$ term. This expression gives the energy to leading order in $\tilde{z}_0$. From equation (4.7), we can then write

$$J \simeq \frac{E\tilde{\rho}^2_{\rm st.}(0)}{\omega} = \frac{R^2\mathcal{C}}{\pi\alpha'\tilde{z}_0} + \mathcal{O}(\tilde{z}_0) \tag{4.31}$$

for the spin. By eliminating $\tilde{z}_0$ and restoring m_q, we finally arrive at

$$E = 2m_q - E_b \,, \tag{4.32}$$

where

$$E_b = m_q \frac{\kappa^4}{4J^2} \,, \qquad \kappa^4 = \frac{16\mathcal{C}^4 g_s N}{\pi} \,. \tag{4.33}$$

It is remarkable that the binding energy E_b coincides exactly with that of a classical system consisting of two non-relativistic particles of equal masses m_q bound by a Coulomb potential $V(\rho) = -\kappa^2/\rho$, the strength of which, κ^2, is precisely that of the static quark-anti quark potential (for large ρ) given by the 'hanging string' calculation of next section.

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Actually, one can see that the dependence of E, J and the radius of the orbit also agree with the classical result. For example, the radius of the orbit is $\tilde{\rho}_0 \simeq \tilde{\rho}_{\rm st.}(0) = \mathcal{C}\tilde{z}_0$, or in terms of the rescaled radius: $\rho_0 = \mathcal{C}\tilde{z}_0/\omega$. The relation $\omega \sim \tilde{z}_0^3$ derived in equation (4.27) gives $\rho_0 \sim \omega^{-2/3}$, namely Kepler's law, that the cube of the radius is proportional to the square of the period of the orbit. Note also that in the classical orbit calculation, a non-relativistic treatment is justified, since the speed v of the quarks, as follows from identifying $E_b \sim m_q v^2$, is $v \sim \sqrt{g_s N}/J \ll 1$.

We would like to emphasize that this agreement is a consequence of a highly non-trivial modification of the open string spectrum on the D7-brane in AdS_5 for large J. This means that a string in AdS space can not only describe the statics of a Coulomb potential [6] but also the dynamics of masses bound by it. It is interesting to see that, in the string calculation, the mass of the quarks, their kinetic energy and the Coulomb energy all come from the energy of the string.

To summarize, from the viewpoint of the boundary theory our results imply that the meson mass, as a function of J, follows a Regge trajectory for small J, whereas for large J it is well explained by particles moving in a Coulomb potential. In the language of QCD, one could say that for small J the quarks behave as light quarks since they are ultra-relativistic, whereas for J large they behave as heavy quarks, i.e., non-relativistically. However, one should not take the analogy with QCD much further because the potential at large distances in our case is not confining, but Coulombic.

5. Quark-antiquark potential

In this section we will compute the *static* potential between a dynamical quark-antiquark pair and we will verify that the result is in precise agreement with the *dynamical*, rotating-string calculation above. Since we closely follow the calculation in reference [6], performed in euclidean signature, we write the relevant part of the AdS_5 metric as

$$ds^2 = \alpha' \left[\frac{U^2}{\mathcal{R}^2} \left(dt^2 + d\rho^2 \right) + \frac{\mathcal{R}^2}{U^2} dU^2 \right] . \tag{5.1}$$

The coordinate U has dimensions of (length)$^{-1}$ and is related to the radial coordinate r of equation (2.1) by $U = r/\alpha'$, while $\mathcal{R}^2 = R^2/\alpha' = \sqrt{4\pi g_s N}$. The D7-brane sits at $U = L/\alpha' = 2\pi m_q$.

We consider a string whose embedding is specified as $U = \mathrm{U}(\rho)$ and whose endpoints lie on the D7-brane at a distance $2\rho_0$ from one another. Without loss of generality, we can picture the string as straddling the point $\rho = 0$, about which the embedding profile is symmetric. Then, choosing a parametrization $t = \tau$, the string action per unit time, namely its energy, takes the form

$$E = \frac{1}{\pi} \int_0^{\rho_0} \mathrm{d}\rho \sqrt{U'^2 + \frac{U^4}{\mathcal{R}^4}} \,, \tag{5.2}$$

where the prime denotes differentiation with respect to ρ. That the lagrangian is independent of ρ implies

$$\frac{U^4}{\sqrt{U'^2 + \frac{U^4}{\mathcal{R}^4}}} = \mathcal{R}^2 U_0^2 \;, \tag{5.3}$$

where $U_0 = \mathrm{U}(0)$ is the minimum value of $\mathrm{U}(\rho)$ and is determined by the condition

$$\rho_0 = \frac{\mathcal{R}^2}{U_0} \int_1^{2\pi m_q/U_0} \frac{\mathrm{d}y}{y^2\sqrt{y^4-1}} \;. \tag{5.4}$$

Therefore, using equation (5.3), the energy (5.2) of the quark-antiquark pair is given by

$$E = \frac{U_0}{\pi} \int_1^{2\pi m_q/U_0} \mathrm{d}y \, \frac{y^2}{\sqrt{y^4-1}} \;. \tag{5.5}$$

We will now see that the two limiting cases of large and small angular momentum discussed above correspond to the quantity $\rho_0 m_{\mathrm{gap}}$ being large and small, respectively — i.e., to large and small separation between the quarks, as compared to the scale set by the lightest meson.

5.1 Large separation

Let us think of keeping ρ_0 fixed and making m_q/U_0 large. We shall see below that this corresponds to the desired, large-$\rho_0 m_{\mathrm{gap}}$ regime. Then we can rewrite (5.4) as

$$\rho_0 = \frac{\mathcal{R}^2}{U_0}\mathcal{C} - \frac{\mathcal{R}^2}{U_0} \int_{2\pi m_q/U_0}^{\infty} \mathrm{d}y \, \frac{1}{y^2\sqrt{y^4-1}} \,, \tag{5.6}$$

where $\mathcal{C}$ was defined in (4.26), and approximate the integrand in (5.6) by y^{-4} to obtain

$$\rho_0 \simeq \frac{\mathcal{R}^2}{U_0}\mathcal{C} - \frac{\mathcal{R}^2 U_0^2}{3(2\pi m_q)^3} \,. \tag{5.7}$$

Solving iteratively for U_0, we find

$$U_0 \simeq \frac{\mathcal{C}\mathcal{R}^2}{\rho_0}\left(1 - \frac{\mathcal{C}^2\mathcal{R}^6}{3\left(2\pi\rho_0 m_q\right)^3}\right) . \tag{5.8}$$

Now, using the leading term of this expression, we find that

$$\frac{m_q}{U_0} \simeq \frac{m_q \rho_0}{\mathcal{C}\mathcal{R}^2} \sim \rho_0 m_{\mathrm{gap}} \,, \tag{5.9}$$

recalling that $m_{\mathrm{gap}} \sim m_q/\sqrt{g_s N}$. Therefore, in the $\rho_0 m_{\mathrm{gap}} \to \infty$ limit, our approximation of the above integral is valid. To compute the energy we first rewrite equation (5.5) as

$$E = 2m_q + \frac{U_0}{\pi}\left[\int_1^{\infty} \mathrm{d}y \left(\frac{y^2}{\sqrt{y^4-1}} - 1\right) - 1\right] - \frac{U_0}{\pi}\int_{2\pi m_q/U_0}^{\infty} \mathrm{d}y \left(\frac{y^2}{\sqrt{y^4-1}} - 1\right) . \tag{5.10}$$

The quantity in square brackets is equal to $-\mathcal{C}$. Approximating the integrand in the last term by $(2y)^{-4}$, with the same justification as above, and using equation (5.8), we arrive at

$$E \simeq 2m_q - \frac{\kappa^2}{2\rho_0}\left(1 - \frac{1}{6}\frac{\mathcal{C}^2\mathcal{R}^6}{(2\pi\rho_0 m_q)^3}\right) , \tag{5.11}$$

where the constant κ (defined in (4.33)) that controls the strength of the dominant, Coulomb-like term in the potential, is exactly the same as that found in [6].[10] As expected, it is also the strength of the Coulomb potential found to bind the slowly-spinning quark-antiquark pair of the previous section. Note that, for fixed 't Hooft coupling and sufficiently large distances, corrections to the Coulomb-like term are suppressed by a cubic power of $1/\rho_0 m_{\rm gap}$. From the field theory viewpoint this can be understood as the fact that, for distances ρ_0 much larger than the scale $m_{\rm gap}^{-1}$ set by the lightest meson, interactions are essentially due only to the exchange of the massless fields in the $\mathcal{N}=4$ multiplet, which must give rise to a Coulomb potential [6].

5.2 Small separation

Let us now think of keeping $m_{\rm gap}$ fixed while decreasing ρ_0. We see from equation (5.4) that $U_0 \to 2\pi m_q$ from below as $\rho_0 \to 0$. In this limit we can set $y = 1+z$, with $z \sim 0$, and approximate equation (5.4) as

$$\rho_0 \simeq \frac{\mathcal{R}^2}{U_0} \int_0^{2\pi m_q/U_0 - 1} \mathrm{d}z\, \frac{1}{\sqrt{4z}} = \frac{\mathcal{R}^2}{U_0} \sqrt{\frac{2\pi m_q}{U_0} - 1}\,. \tag{5.12}$$

Similarly, the expression for the energy, equation (5.5), becomes

$$E \simeq \frac{U_0}{\pi} \sqrt{\frac{2\pi m_q}{U_0} - 1}\,. \tag{5.13}$$

Following our previous strategy, we can solve iteratively for U_0 from equation (5.12) to obtain

$$U_0 \simeq 2\pi m_q \left(1 - \frac{(2\pi\rho_0 m_q)^2}{\mathcal{R}^4}\right), \tag{5.14}$$

and substitute this into the energy (5.5) to find

$$E \simeq \frac{4\pi\rho_0 m_q^2}{\mathcal{R}^2} = (2\rho_0)\tau_{\rm eff.}\,. \tag{5.15}$$

Therefore, for small quark-antiquark separation, $2\rho_0$, there is a string-like (linear) potential with an effective tension $\tau_{\rm eff.} = m_q^2\sqrt{\pi/g_s N}$, precisely the same as that found to determine the energy of the rapidly-rotating string discussed in the previous section (see equation (4.19)). The next-to-leading term in the energy is suppressed by a factor of $(\rho_0 m_{\rm gap})^2$ with respect to the leading term above.

Note that this linear potential, at short distances, is weaker than the Coulomb potential (see figure 3). The fact that the transition between the two behaviours occurs at a distance of order $m_{\rm gap}^{-1}$ suggests that this 'screening' is caused by the meson-exchange contribution to the potential.

[10] Since the calculation in this reference was done for external, infinitely heavy quarks, the rest mass $2m_q$ of the quark-antiquark pair was subtracted in order to regularize the result.

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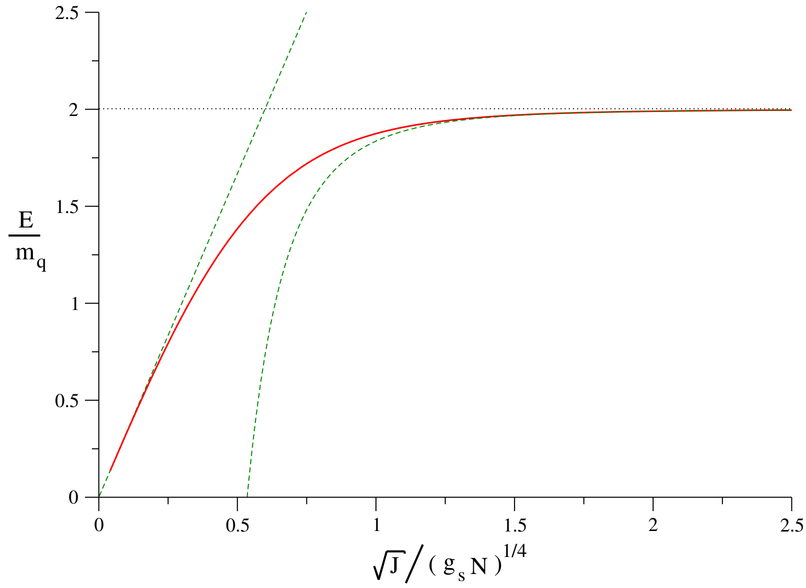

Figure 3: Meson spectrum $E(J)$ obtained numerically. The horizontal line represents the rest mass of the quark-antiquark pair. The other dashed, straight line is a plot of the Regge behaviour (4.18) exhibited by the spectrum for $J \ll \sqrt{g_s N}$. Finally, the curved dashed line is a plot of $E(J)$ as given by equations (4.32) and (4.33), which represents the energy of two non-relativistic quarks bound by a Coulomb potential and describes the spectrum well in the limit $J \gg \sqrt{g_s N}$.

6. Baryons

The $\mathcal{N} = 4$ SU(N) SYM theory does not contain matter in the fundamental representation, so there are no dynamical baryons. There is, however, a baryon vertex, that is, a gauge-invariant, antisymmetric combination of N external charges. In the dual string theory description this is represented by a D5-brane wrapped on the five-sphere at some AdS radius r and connected to the AdS boundary by N fundamental strings [12]. The energy of this system is infinite because of the infinite length of the strings [21]. Each string defines a 'direction' in $AdS_5 \times S^5$. If all strings are oriented in the same direction, then this configuration preserves half of the sixteen background Poincaré supersymmetries, regardless of the common string direction and of the radial position r of the D5-brane [21].[11] This is consistent with the conformal invariance of the gauge theory, which implies that there cannot be any preferred size for the baryon vertex.

Consider introducing the D7-brane. In the gauge theory we now have $\mathcal{N} = 2$ supersymmetry and fundamental quarks, so we expect dynamical, finite-energy, supersymmetric baryons to exist. It is not hard to see what the dual of such an object must be — a D5-brane wrapped on the five-sphere and connected by N fundamental strings to the D7-brane. One can imagine constructing it as follows. Start with an infinitely heavy baryon, i.e., with a D5-brane connected by N strings to the AdS boundary. If the strings are oriented in such a way that they intersect the D7-brane then they can each break into two pieces, one connecting the D5- to the D7-brane and one connecting the D7-brane to the AdS bound-

[11]There are also sixteen special conformal supersymmetries in the $AdS_5 \times S^5$ background that we will disregard in this discussion because they are all broken by the D5-brane.

ary. These second pieces can then be 'moved away' leaving behind the dynamical baryon. This corresponds to a process in the gauge theory in which, for example, a bound state of N infinitely heavy quarks Q splits into a bound state of N dynamical quarks q and N infinitely heavy mesonic $Q\bar{q}$ bound states (dual to the string pieces from the D7-brane to the AdS boundary).

Thinking of constructing the dynamical baryon from the baryon vertex in this way seems to imply that it can lie at an arbitrary radial position in AdS because the initial baryon vertex certainly can. This contradicts the field theory expectation that there should be a preferred size for the dynamical baryon now that a mass scale m_q has been introduced. Presumably the resolution lies in the fact that the construction above ignores the precise way in which the strings are attached to the D7-brane. Once this is taken into account, the radial position of the D5-brane should no longer be a free parameter. This would be analogous to the fact that the size of a monopole realized as a spike connecting two parallel D3-branes is related to the distance between the branes. In fact, one can think of realizing the baryon itself as an BIon-like [20] excitation of the D7-brane: one can imagine it as a spike, representing the N strings, that emanates from the D7-brane and subsequently closes upon itself forming the (hemi)spherical D5-brane. The strings and the D5-brane would in this way be constructed 'out of the fields on the D7-brane', being associated to scalar excitations, as well as to local non-zero electric (for the strings) and magnetic (for the D5-brane) fluxes of the gauge field on the D7-brane. Since the D7-brane fields are in turn associated to meson states in the gauge theory, this picture would provide the string realization of the baryon as a bound state of mesons, as in the skyrme model [19]. One reason that makes an explicit construction of the relevant solution of the D7-brane worldvolume theory difficult is that the spike should interpolate between a D7- and a D5-brane, so its topology will be complicated.

The fact that the dynamical baryon is supersymmetric is not completely obvious because one may worry that the supersymmetry preserved by the spherical D5-brane might be incompatible with that preserved by the D7-brane. A simple way to see that this is not the case is to recall why supersymmetry is preserved by the infinitely heavy baryon vertex [21]. Each of the three elements in this system, namely, the D3-branes creating the AdS background, the spherical D5-brane and the N parallel strings, imposes a projection on the preserved supercharges that, acting individually, halves their number. However, only two of these projections are independent, hence eight supercharges (as opposed to only four) are preserved. Since we may choose the two independent projections to be those associated with the D3-branes and to the strings, it is clear that the introduction of the D7-brane will just halve again the total amount of supersymmetry, as long as it is parallel to the D3-branes and orthogonal to the strings. We thus conclude that the dynamical baryon vertex preserves one-half of the supersymmetry of the $\mathcal{N}=2$ gauge theory.

7. Discussion

Introducing a finite number of D7-brane probes in $AdS_5 \times S^5$ is dual to introducing a finite number of flavours (i.e., hypermultiplets in the fundamental representation of the gauge group) in the $\mathcal{N}=4$ SYM theory [1]. The resulting system enjoys $\mathcal{N}=2$ supersymmetry.

In this paper we have used the bulk side of this AdS/CFT correspondence with flavour to analyze in detail the spectrum of field theory mesons. These are quark-antiquark bound states, which come in supersymmetry multiplets and hence can be bosonic or fermionic. Their bulk duals are open strings with both ends on the D7-brane. The strength of their interactions is given by the open string coupling constant, which scales as $g_o \sim 1/\sqrt{N}$ for large-N (with $g_s N$ fixed). Closed string interactions scale as $g_s = g_o^2 \sim 1/N$, as corresponds to glueball interactions in the large-N gauge theory. The string picture thus provides a particularly simple understanding of large-N field theory scalings.

To compute the meson spectrum one should quantize these open strings. Since that is not possible at the moment we have resorted to a field theory approximation (i.e., to using the D7-brane DBI action) for mesons with R-charge, which are dual to D7-brane massless modes with angular momentum on the five-sphere. For mesons with (large) four-dimensional spin, which are dual to highly-excited rotating strings, we have used a semi-classical approximation.

In the first case we found the spectrum by solving the equations of motion obtained from the quadratic approximation to the DBI action. This calculation is similar to what was done in the literature for glueball masses in the supergravity duals of confining theories [24]. However, here we have solved the equations analytically in terms of hypergeometric functions and have obtained the spectrum in closed form. It possesses a mass gap of order $m_q/\sqrt{g_s N}$, which shows that the mesons are much lighter than the quarks in the large-'t Hooft coupling limit. Therefore, together with the $\mathcal{N} = 4$ vector multiplet, they dominate the low-energy dynamics.

One interesting feature is the presence in the spectrum of massive vector mesons that couple universally to the rest of the mesons. This is similar to what happens with the ρ-meson in QCD. In our case the universality of the coupling is due to the gauge invariance of the eight-dimensional D7-brane action, whose dimensional reduction gives the four-dimensional effective meson lagrangian. The gauge vector meson is nothing else than the dimensional reduction of the gauge field on the D7-brane. In QCD the universality of the coupling has also been attributed to a hidden gauge symmetry [22] and it would be interesting to see if these two ideas are related.

The fact that we obtained the meson masses in closed form allowed us to uncover an unexpected classification of the free spectrum in SO(5) representations, even though the theory possesses only a manifest SO(4) $\simeq$ SU(2)$_R$ $\times$ SU(2)$_L$ symmetry. For modes that transform in the same representation of SU(2)$_R$ and SU(2)$_L$ the SO(5) is a symmetry, that is, all states in a given SO(5) multiplet have the same mass. The masses of mesons in different SU(2)$_{L,R}$ representations are not the same, but are related to one another in a very simple way. It would be interesting to analyze this 'symmetry enhancement' from the gauge theory viewpoint.

We noted that the meson radial mode functions peak more and more sharply at $\rho = L$ as the meson R-charge ℓ increases, and that this is precisely the value at which classical, point-like, collapsed, massless open strings can orbit the S^3 along a null geodesic. These are potentially interesting because they are open string analogues of the closed strings recently studied in [15, 11]. The latter are point-like, collapsed, massless closed strings

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that orbit the five-sphere of $AdS_5 \times S^5$ along a null geodesic. The effective geometry seen by these strings is that of an Hpp-wave [25], as was derived in [15] by taking the Penrose limit of $AdS_5 \times S^5$ [26]. It was subsequently observed in [11] that the same result can be obtained by simply expanding the string action for small fluctuations around the collapsed, orbiting solution. The fact that the Penrose limit of a supergravity solution is a solution itself [26] is crucial, because it guarantees the consistency of the quantum theory obtained by quantization of the above quadratic fluctuations of the string. In view of the amount of progress recently made by studying Penrose limits of supergravity spacetimes with field theory duals, it would clearly be interesting to understand the dynamics of quadratic fluctuations of open strings around the S^3 geodesics above. Note that these are geodesics in the induced metric on the D7-brane, but *not* in the $AdS_5 \times S^5$ geometry. Therefore the result will not simply be equivalent to some Penrose limit, so quantum-mechanical consistency of the quadratic approximation is not guaranteed.

As well as point-like behaviour, we expect that the large R-charge open string states should exhibit expanded brane-like behavior, just as in the closed string sector. We begin by addressing this issue in the massless quark limit $L \to 0$. Then one expects a direct correspondence between single-particle supergravity and D7-brane BI modes in the $AdS_5 \times S^5$ background and chiral primaries in the dual gauge theory. With the set-up described in sections 1 and 2, the operators $\mathrm{Tr}\, \mathcal{X}_\ell$ defined in equation (3.74) would describe closed string states carrying total angular momentum ℓ on the S^3 of the D7-brane. For example, a closed string orbiting the S^3 along a great circle in the 45-plane, say, would be described by the operator $\mathrm{Tr}\, Z^\ell$, where $Z = \Phi^1 + i\Phi^2$. As is well known, however, at finite N these operators are only independent for $\ell = 1, \ldots, N$ because of the finite rank of the gauge group. The truncation of this family of operators at $\ell = N$ is manifest in the AdS bulk through the appearance of giant gravitons [27], i.e., expanded, spherical D3-branes following a trajectory on the above circle. Of course, the precise gauge theory description of these configurations is more intricate than implied above, involving certain subdeterminant operators [28] that implicitly extend the above through the addition of a combination of multi-trace operators.

Consider now the 'hypermultiplet' operators $\mathcal{O}_\ell$, defined in (3.75), dual to gauge field modes on the D7-brane. For large ℓ these also carry angular momentum ℓ on the S^3; the operator $\bar{\phi}^m \sigma_{mn} Z^\ell \phi^n$, for example, would describe open string states orbiting the great circle in the 45-plane. For this family, the operators are only independent for $\ell = 0, \ldots, N-1$ for finite N. This truncation should be realized in the AdS space through the appearance of the same extended D3-brane states as above. These spherical D3-branes would intersect the D7-brane on an circle in the 67-plane. Realizing the above hypermultiplet operators would involve exciting a pair of (7,3) and (3,7) strings on this intersection.[12] The precise gauge theory dual of these 'giant gauge bosons' is presumably closely related to the subdeterminant operators discussed in [28, 30]. A natural conjecture is that one of the adjoint fields $(Z)_a{}^b$ is replaced by the combination $(\bar{\phi}^m)_a \sigma_{mn} (\phi^n)^b$.

[12] Modelling this intersection with the intersection of planar D7- and D3-branes on a line would result in a supersymmetric open string spectrum with massless ground state modes. In the present case, however, where the D3-brane has a finite spatial volume, consistency would require that the open strings are excited in oppositely oriented pairs.

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The truncation of the hypermultiplet operators is certainly unaffected by the quark masses, and so one expects that giant gauge bosons still play a role when $L \neq 0$. The precise description of the AdS configurations is more complicated but presumably involves an expanded, spherical D3-brane connected to the D7-brane with a pair of open strings. It would be interesting to investigate these states in more detail. Studying the dual giant gauge bosons, analogous to the dual giant gravitons [29], may also provide useful insights.

To obtain the spectrum of mesons with large spin J we considered semiclassical, rotating open strings attached to the D7-brane, following the approach of [11] for closed strings. We solved the string equations of motion numerically for arbitrary J, and analytically in the limiting regimes $J \ll \sqrt{g_s N}$ and $J \gg \sqrt{g_s N}$. An important subtlety was that the strings must end orthogonally on the D7-brane, from which it followed that the endpoints move at subluminal speed. This should be contrasted with the analogous case in flat space, in which the string would actually be contained within the D-brane and its endpoints would move at the speed of light. The physical difference between the two cases is that in AdS the 'weight' of the string (that is, the non-trivial background) pulls it away from the D-brane. At the endpoints of the string this force can be compensated by the string tension only if the string ends orthogonally on the D-brane.

The result of the numerical integration was unexpected: for each angular velocity of the string there is a series of solutions distinguished by the number of nodes $n = 0, 1, 2, \ldots$ of the string (see figure 1). The projection on the AdS boundary suggests a structure for the corresponding meson in which the two quarks are surrounded by concentric shells of gluons associated to the pieces of string between each two successive nodes.

We argued that, presumably, the solution with no nodes is the most stable, since a string is prone to breaking at a self-intersection point, a process that in the gauge theory would correspond to the decay of an excited meson via the emission of a gluon shell. We therefore concentrated on nodeless solutions, the spectrum of which is shown in figure 3.

For $J \ll \sqrt{g_s N}$ the string is much shorter than the AdS radius and the spectrum is accordingly that of flat space, i.e., it follows a Regge trajectory. The effective tension, which we computed analytically to be of order $m_q^2/\sqrt{g_s N}$, can be interpreted as a proper tension, $1/2\pi\alpha'$, appropriately red-shifted. A 'hanging string' calculation confirmed that the static quark-antiquark potential in this regime is linear in the quark separation, with an effective tension identical to that found in the dynamical case.

For $J \gg \sqrt{g_s N}$ the spectrum is drastically modified and takes the form $E = 2m_q - E_b$, where $E_b \propto J^{-2}$ is given in (4.33). The form of the binding energy E_b is precisely that of two non-relativistic masses bound by a Coulomb potential. Again this was confirmed by a static 'hanging string' calculation. From the field theory viewpoint this is understood from the fact that in this regime the distance between the quark-antiquark pair is much larger than the inverse mass of the lightest meson. This means that the interactions are almost solely due to the exchange of fields in the $\mathcal{N} = 4$ vector multiplet, which leads to a Coulomb potential [6].

The meson spectrum is summarized in figure 4.

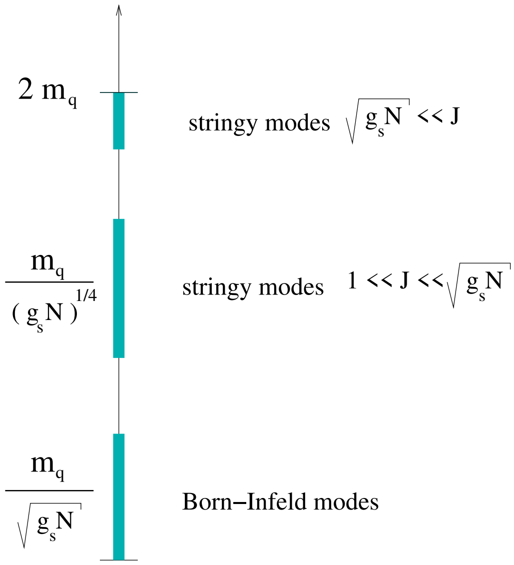

Figure 4: Summary of the meson spectrum; on the right-hand side we have written the type of dual stringy mode.

We would like to remark that the results obtained here are interesting not only for their implications for the dual field theory, but also in themselves because they provide a prediction for the spectrum of open strings on D7-branes in $AdS_5 \times S^5$. Similar results should presumably be obtained for other D-branes in curved backgrounds.

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